Computing the Spectrum

- Objectives
  - Discrete Fourier Transform
  - Spectrum Analysis of Finite-Length and Periodical Signals
  - The Spectrogram
  - FFT
- Reading Assignments
  - Chapter 12 (DTFT and Inverse DTFT), Chapter 13
  - Prepare: Chapter 12 in ‘Digital Signal Processing’

Discrete-Time Fourier Transform

$$X(z)\Big|_{z=e^{j\omega T_s}} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}\Big|_{z=e^{j\omega T_s}} = \sum_{n=-\infty}^{\infty} x[n]e^{j\omega T_s} = X(e^{j\omega T_s})$$

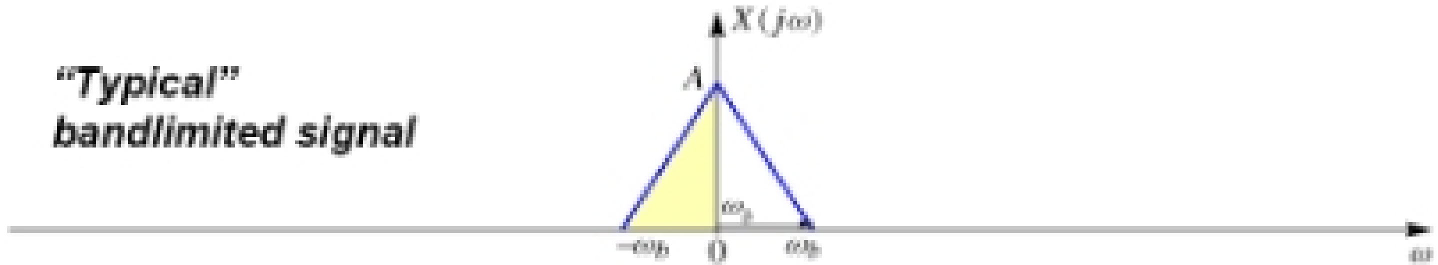
DTFT

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\hat{\omega}n}$$

$$\hat{\omega} = \omega T_s$$

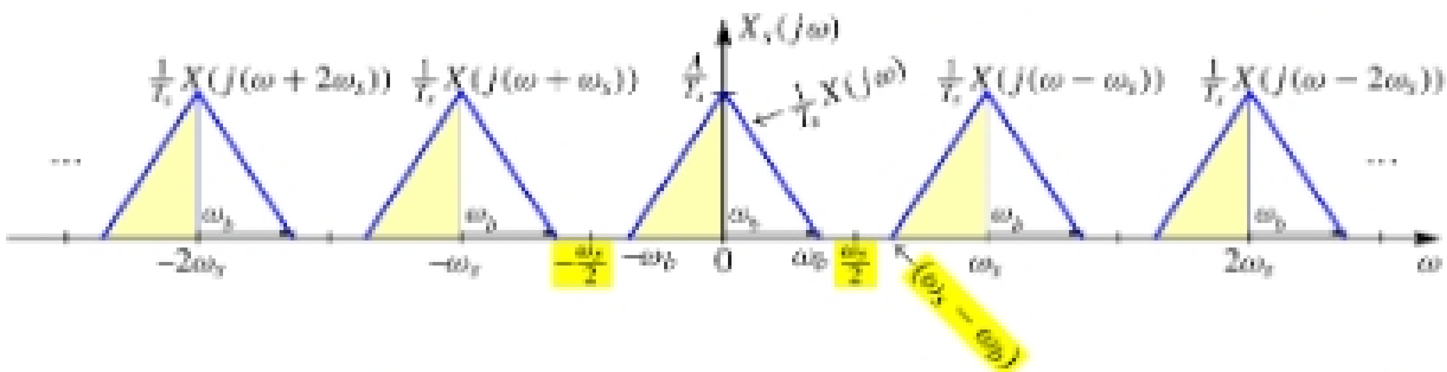
Discrete-Time FT and Continuous-Time FT

"Typical" bandlimited signal



$$x_s(t) = x_c(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s) = x_c(nT_s)\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$$

$$X_s(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$



$$X(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\omega T_s} = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X_c(j(\omega - 2\pi k/T_s))$$