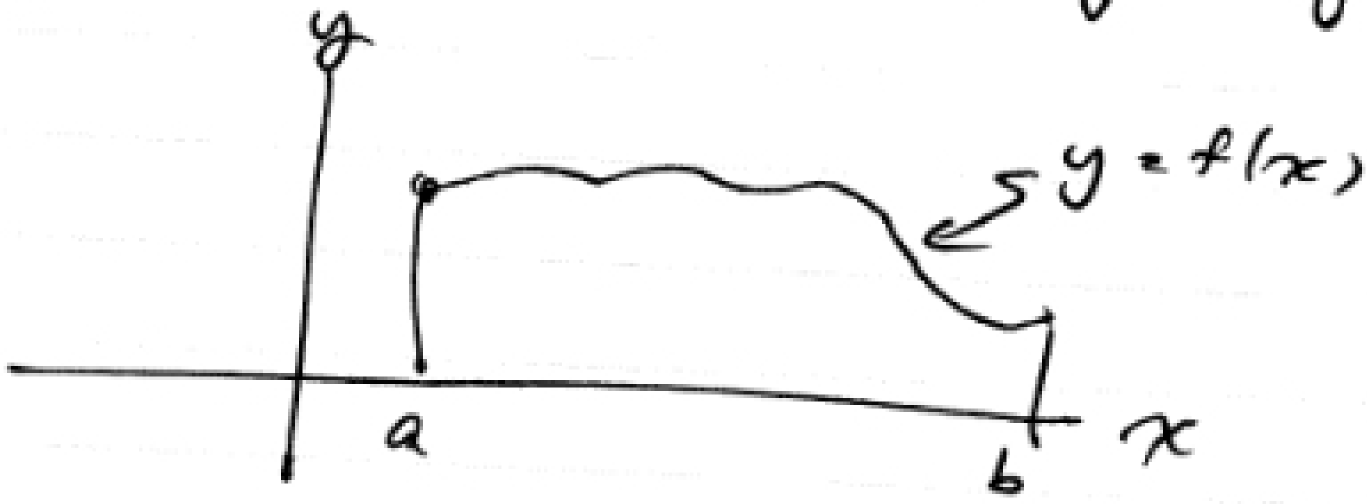
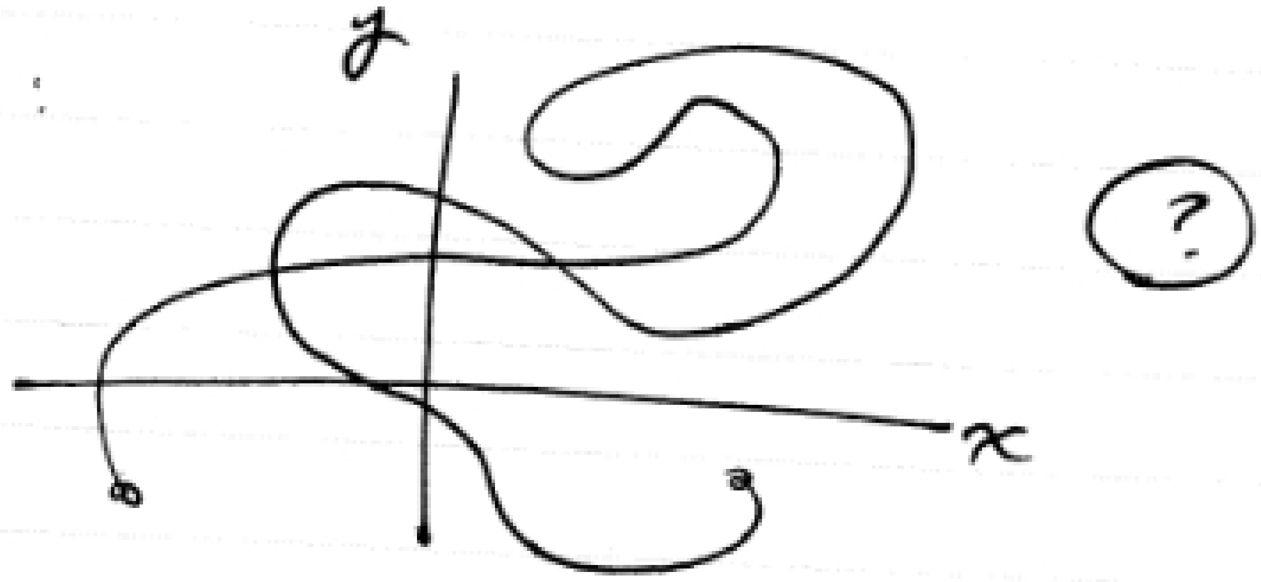
Parametric Equations.

So far, all of our curves have been graphs of functions.

$y = f(x)$



But what about :

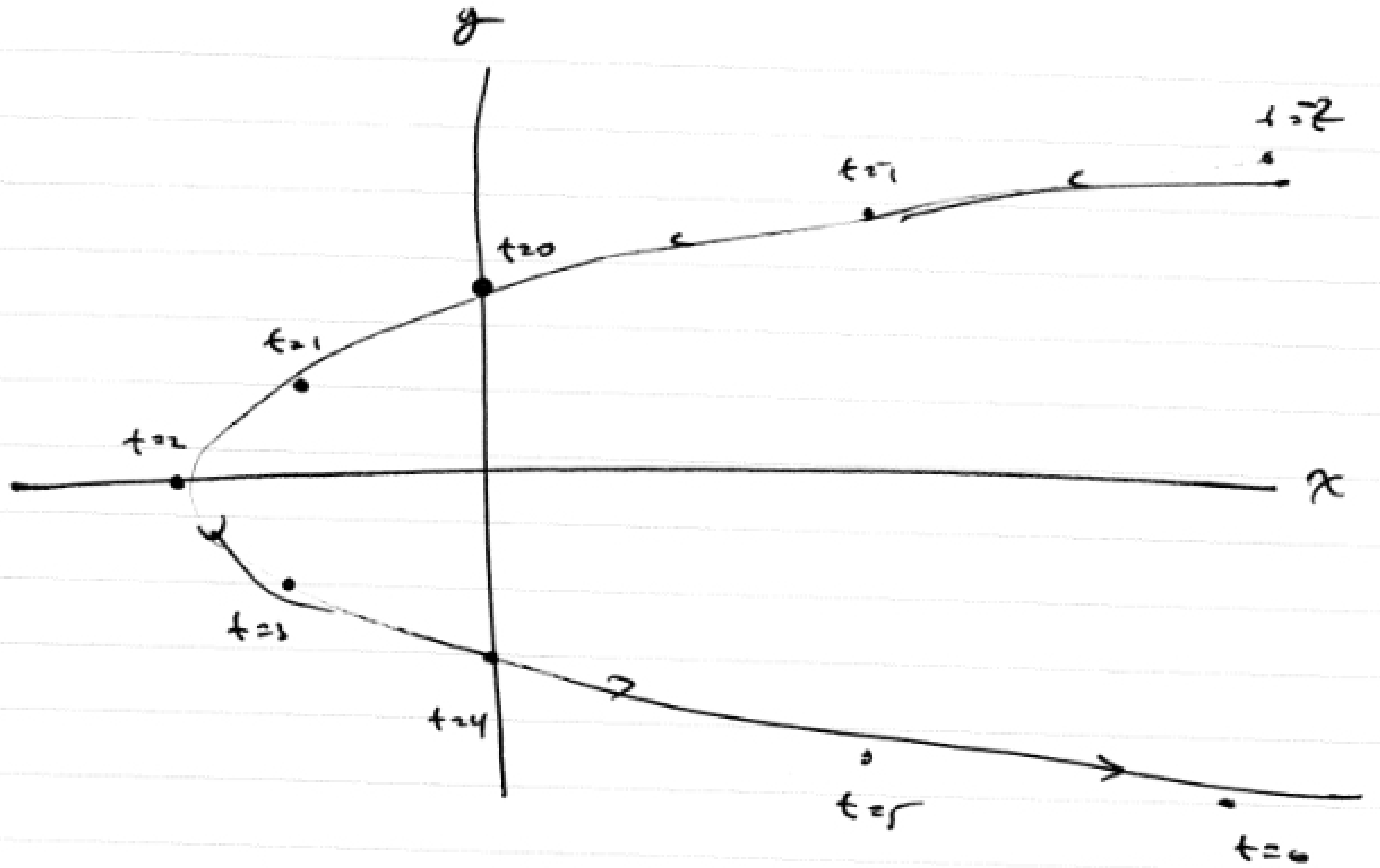


If we write $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ where f, g are functions of another variable t, Then we can do this!

(D) parametric curve.

E.g. $x = t^2 - 4t$
$y = 2 - t$

t	x	y
-2	12	4
-1	5	3
0	0	2
1	-3	1
2	-4	0
3	-3	-1
4	0	-2
5	5	-3
6	12	-4



Why a parabola? $y = 2 - t$

$t = 2 - y$

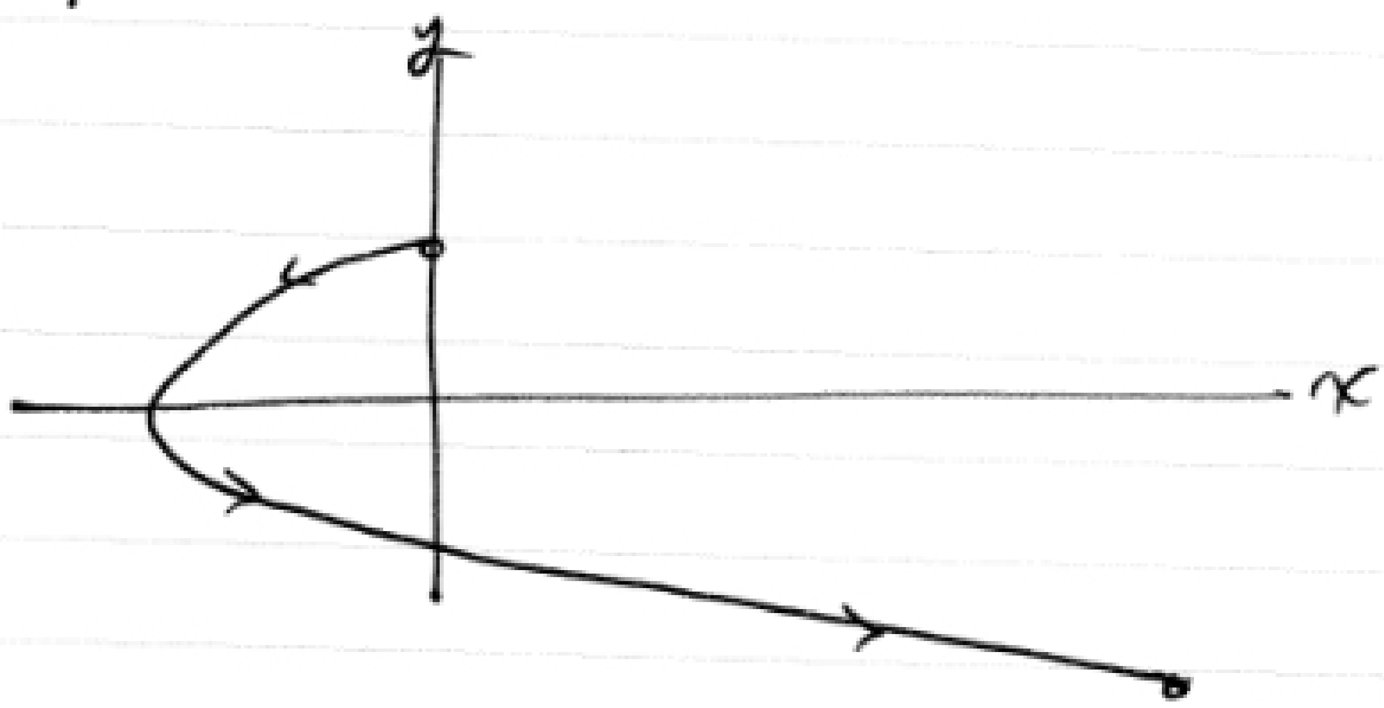
$x = t^2 - 4t = (2-y)^2 - 4(2-y) =$

$= 4 - 4y + y^2 - 8 + 4y =$

$\boxed{x = y^2 - 4}$

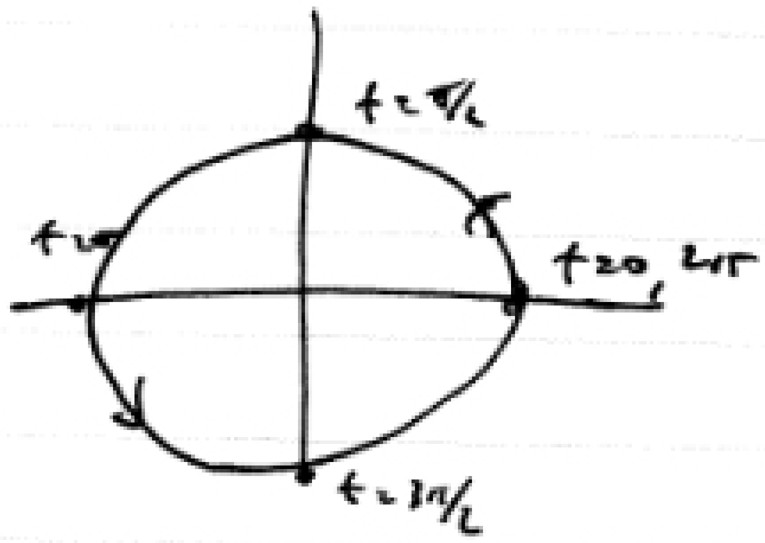
Could also consider

$\left\{ \begin{array}{l} x = t^2 - 4t \\ y = 2 - t \end{array} \right\}$ for $0 \le t \le 6$



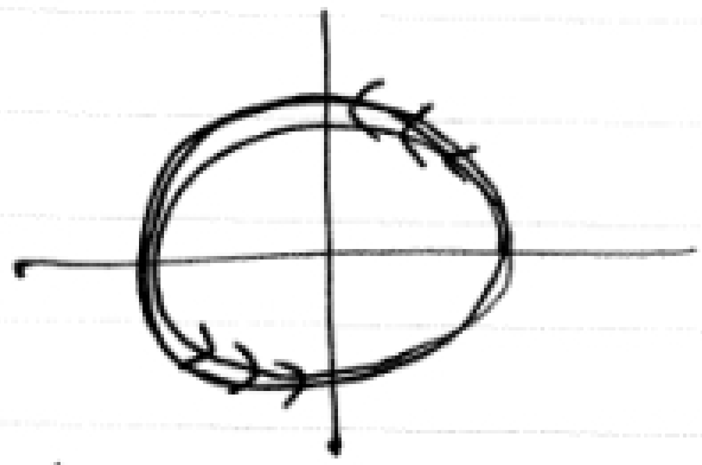
E.g. $x = \cos t$ $\quad 0 \le t \le 2\pi$.
$y = \sin t$

t	0	$\pi/2$	π	$3\pi/2$	2π
x	1	0	-1	0	1
y	0	1	0	-1	0



notice: $x^2+y^2 = \cos^2 t + \sin^2 t = 1$

$x = \cos t$ $\quad 0 \le t \le 6\pi$.
$y = \sin t$



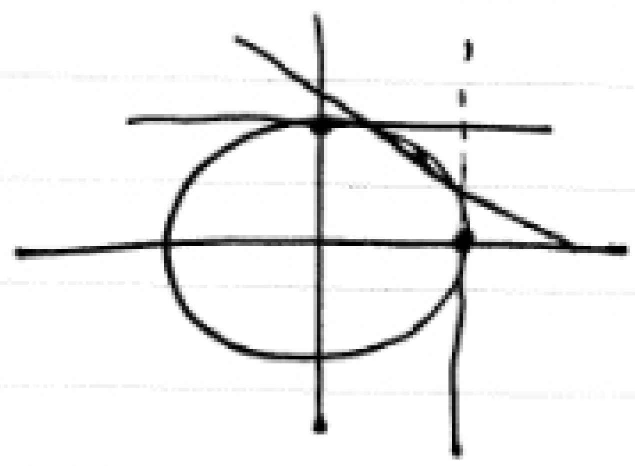
Calculus. Let's say we want the slope of a parametric curve.

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}, \text{ so } \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad (\text{C.R.!})$$

(as long as $dx/dt \neq 0$!)

Go back to circle: $dx/dt = -\sin t$
$dy/dt = \cos t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t} = -\frac{x}{y}.$$



$x = y = \sqrt{2}$. $\quad -1$
$x = 0, y = 1$ $\quad 0$
$x = 1, y = 0$ $\quad$ undefined!