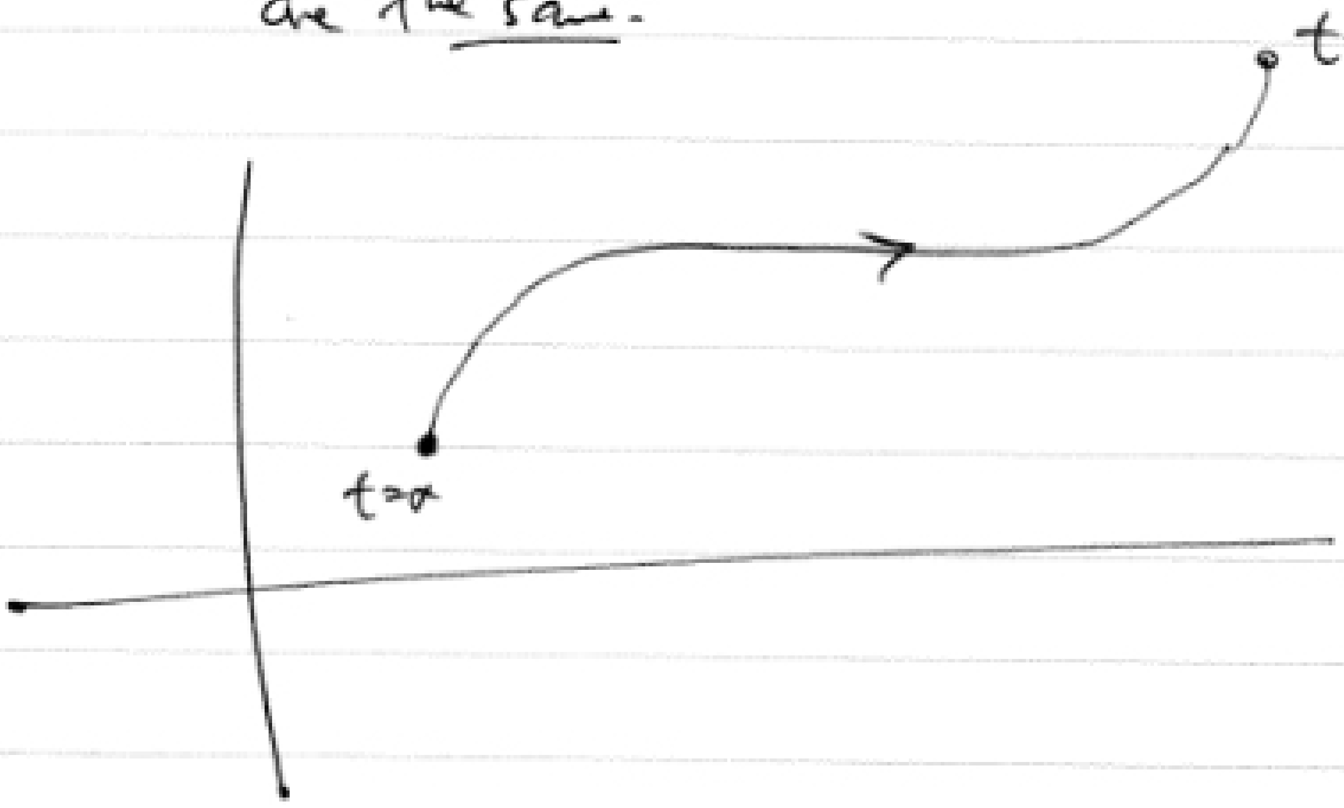
Review 14B4,5,6.

Notice! arclength of

$x = \cos 3t$, $y = \sin 3t$, $t \in [0, 2\pi]$ and $x = \cos t$, $y = \sin t$, $t \in [0, 6\pi]$

are the same.



$x = f(t)$ $\quad \frac{dx}{dt} = f'$

$y = g(t)$ $\quad \frac{dy}{dt} = g'$

$\alpha \le t \le \beta$.

Rescale time

$x = f(2t)$

$y = g(2t)$

$\frac{\alpha}{2} \le t \le \frac{\beta}{2}$.

$\frac{dx}{dt} = 2f'(2t)$

$\frac{dy}{dt} = 2g'(2t)$

$\frac{dy}{dx} = \frac{2g'(2t)}{2f'(2t)}$

Curve is the same!

Arrow points in same direction.

But tangent vector is twice as long!

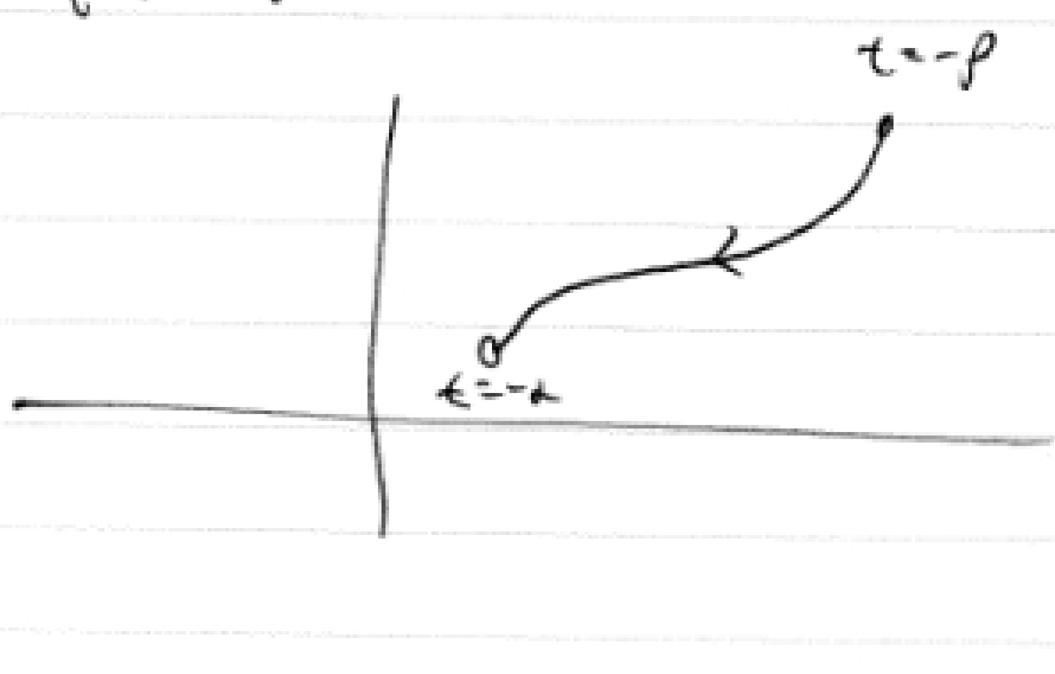
We say "traverse 2nd curve twice as fast."

What if we send $t \to -t$?

$x = f(-t)$

$y = g(-t)$

$-\beta \le t \le -\alpha$



Tangency $\quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

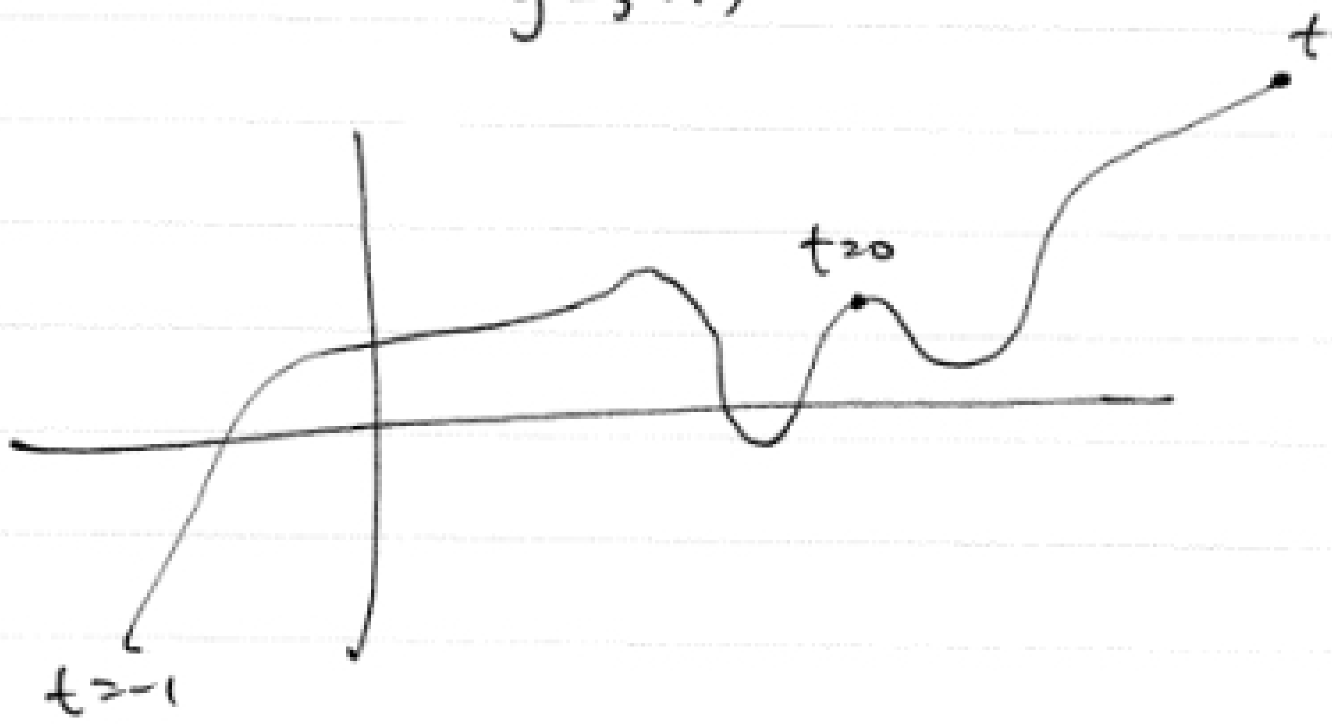
If $\frac{dy}{dt}, \frac{dx}{dt} \neq 0$, perfectly fine, get non-zero #.

$\frac{dy}{dt} = 0, \quad \frac{dx}{dt} \neq 0$ — horizontal tangency

$\frac{dx}{dt} = 0, \quad \frac{dy}{dt} \neq 0$ — vertical tangency

$\frac{dx}{dt} = 0, \quad \frac{dy}{dt} = 0$ — anything can happen!

For example, let $x = f(t)$, $y = g(t)$, $-1 \le t \le 2$ be any finite curve



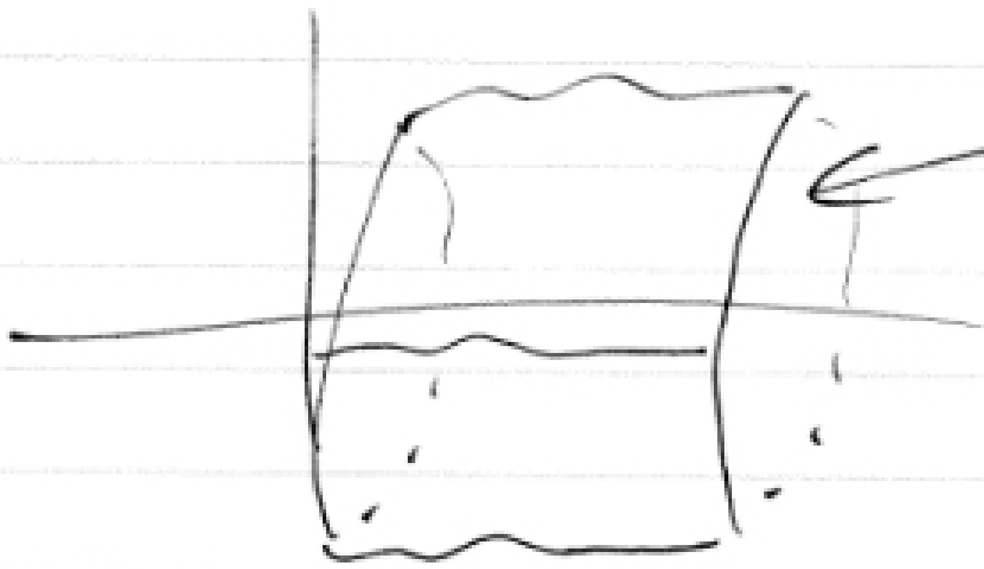
Say $f'(0) \neq 0$
$g'(0) \neq 0$

New curve: $x = f(t^3)$, $y = g(t^3)$ $\quad \frac{dx}{dt} = 3t^2 f'(t^3)$, $\frac{dy}{dt} = 3t^2 g'(t^3)$

$\left.\frac{dx}{dt}\right|_{t=0} = 0 \quad \left.\frac{dy}{dt}\right|_{t=0} = 0.$ $\quad \frac{dy/dt}{dx/dt}$ undefined

Surface Area.

surface area?

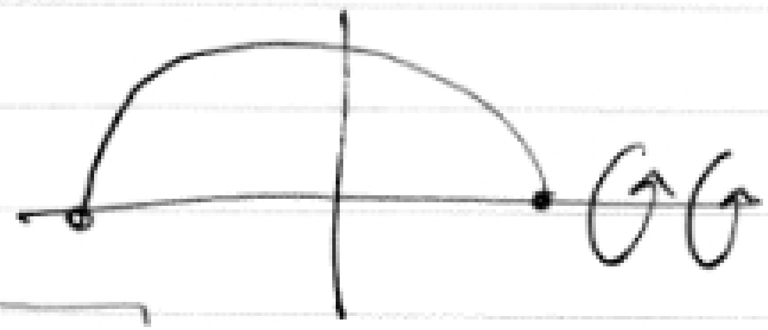


$\int 2\pi y \, ds$

$$= \int_\alpha^\beta 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$x = r\cos t \quad t \in [0,\pi]$

$y = r\sin t$



$$\int_0^\pi 2\pi (r\sin t) \sqrt{r^2\cos^2 t + r^2 \sin^2 t} \, dt$$

$$= 2\pi r^2 \int_0^\pi \sin t \, dt$$

$$(2\pi r^2) \left\{ \left[-\cos t\right]_0^\pi = -(-1) - (-(1)) = 1 + 1 = 2 \right\}$$

$4\pi r^2$