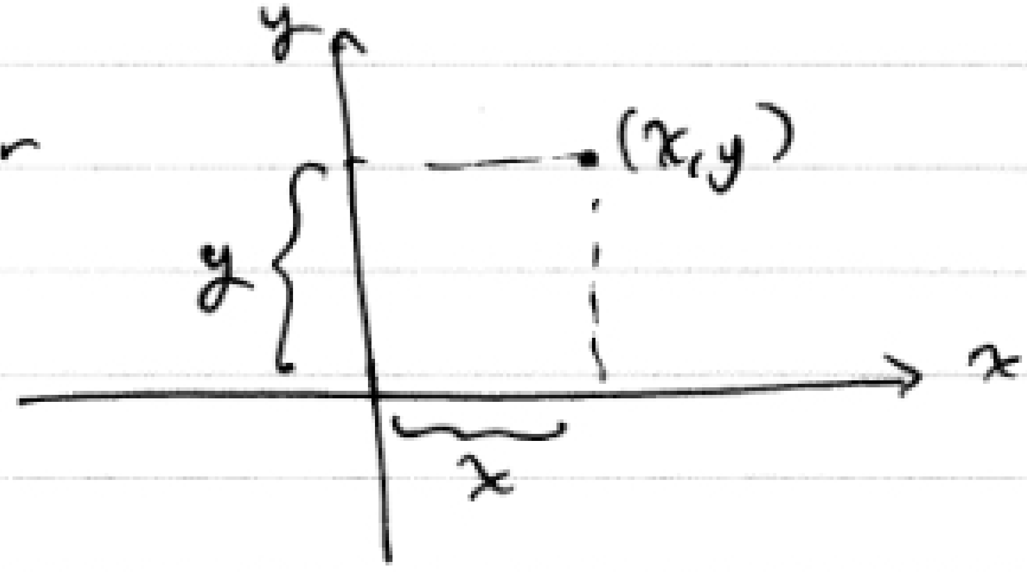
Polar coordinates.

Def. Given two numbers $x, y \in \mathbb{R}$, we associate them to a point on the plane.

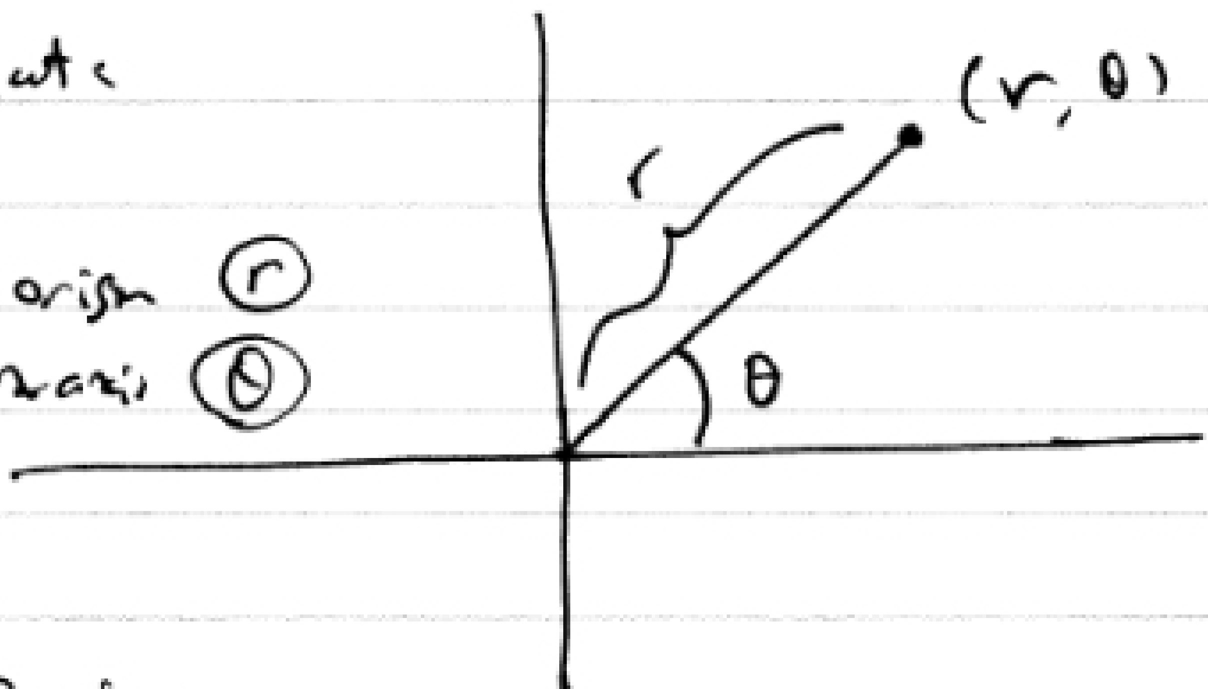
These are called Cartesian or rectangular coordinates.



Another way to represent:

compute distance from origin r

compute angle with positive x-axis θ



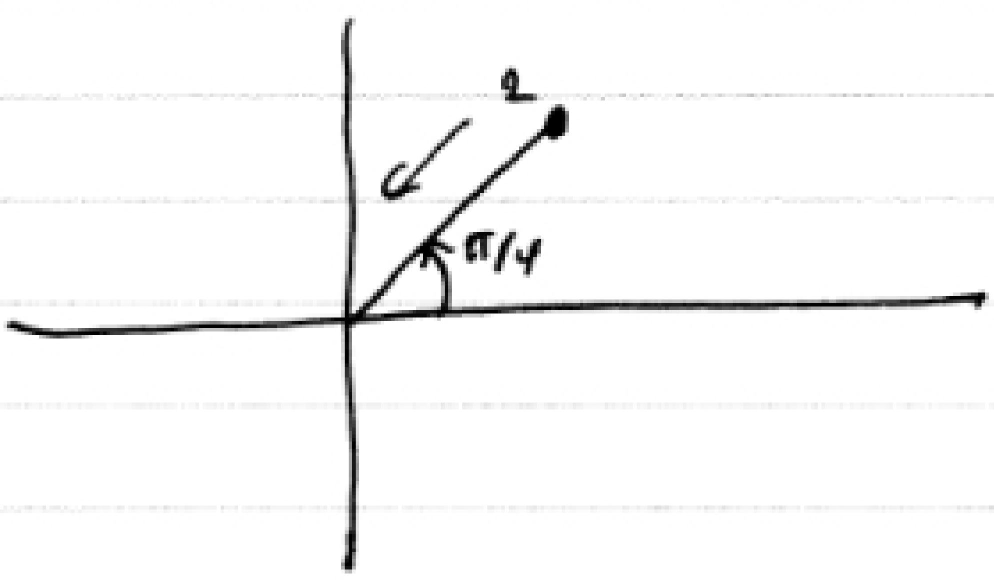
~~For any point except origin,~~

choose any $r \geq 0$, $\theta \in [0, 2\pi)$, this gives point on plane.

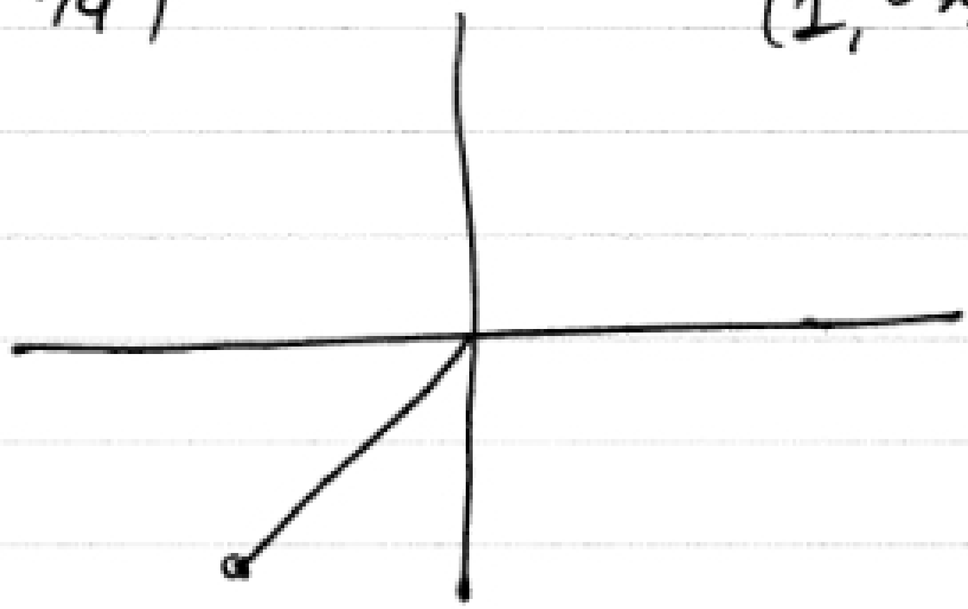
Conversely, given point in plane, this gives (r, θ)

but θ is not unique!

$(1, \pi/4)$



$(1, 5\pi/4)$



We also allow for $r<0$, just go in other direction.

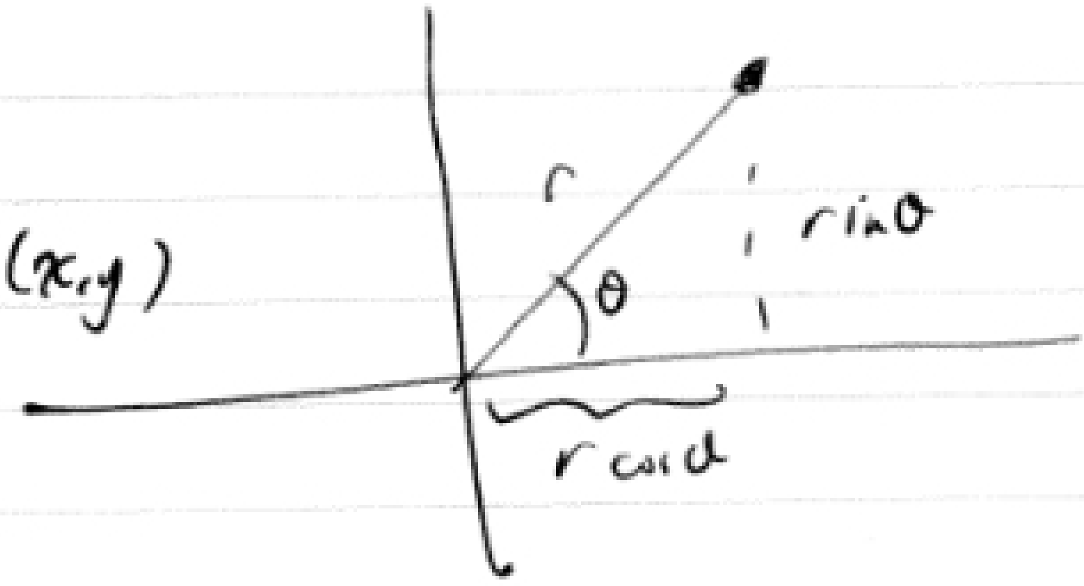
i.e. if $r<0$, then $(r,\theta) \overset{\text{in polar}}{=} (|r|, \theta+\pi)$.

<u>converting</u>:

$x = r\cos\theta$

$y = r\sin\theta$

$(r,\theta) \mapsto (x,y)$



Given x, y,

$r^2 = x^2+y^2$

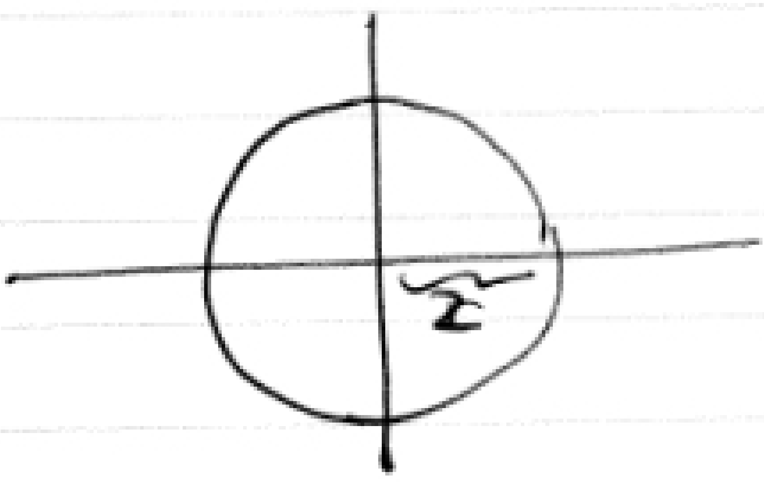
$r = \sqrt{x^2+y^2}$

$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$

Polar <u>curves</u>: we typically write $f(r,\theta)=0$, want to graph it.

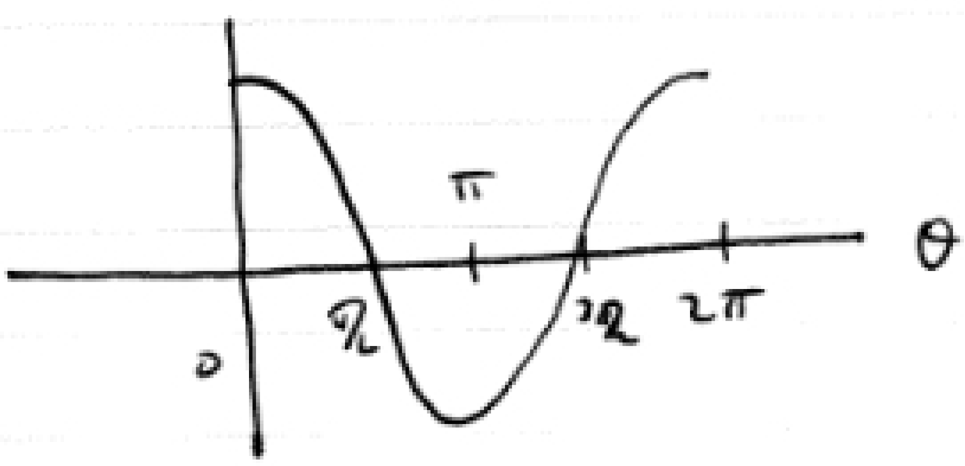
$r = g(\theta)$ is even more common

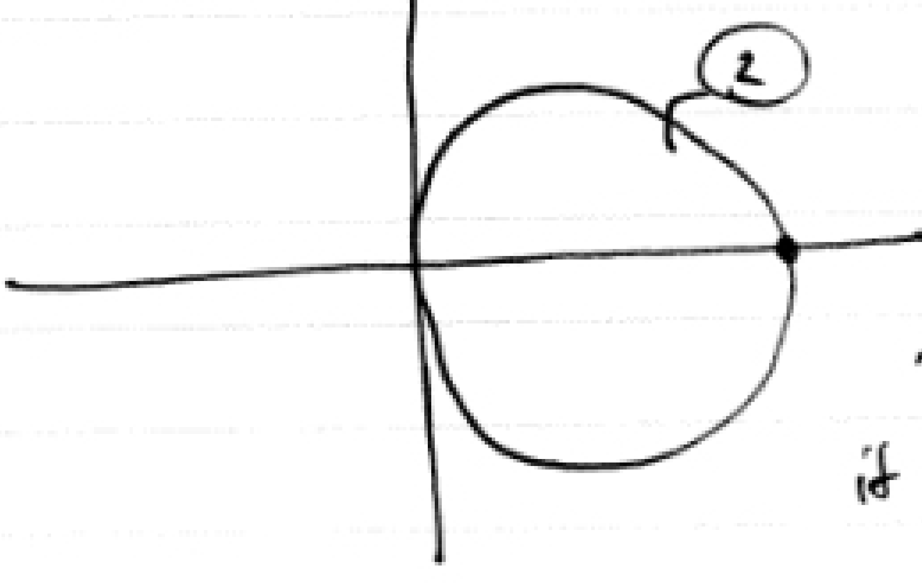
<u>E.g.</u> $r = 2$.



<u>Circle!</u>

E.g. $r = \underline{\cos\theta}$, $\theta \in [0,2\pi]$.





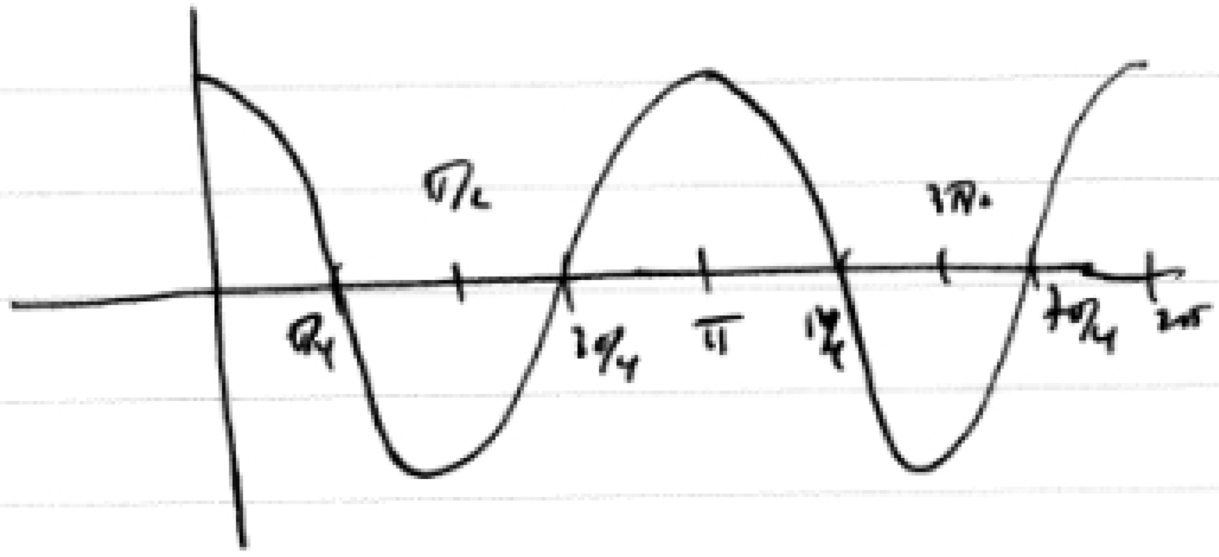
<u>twice!</u>

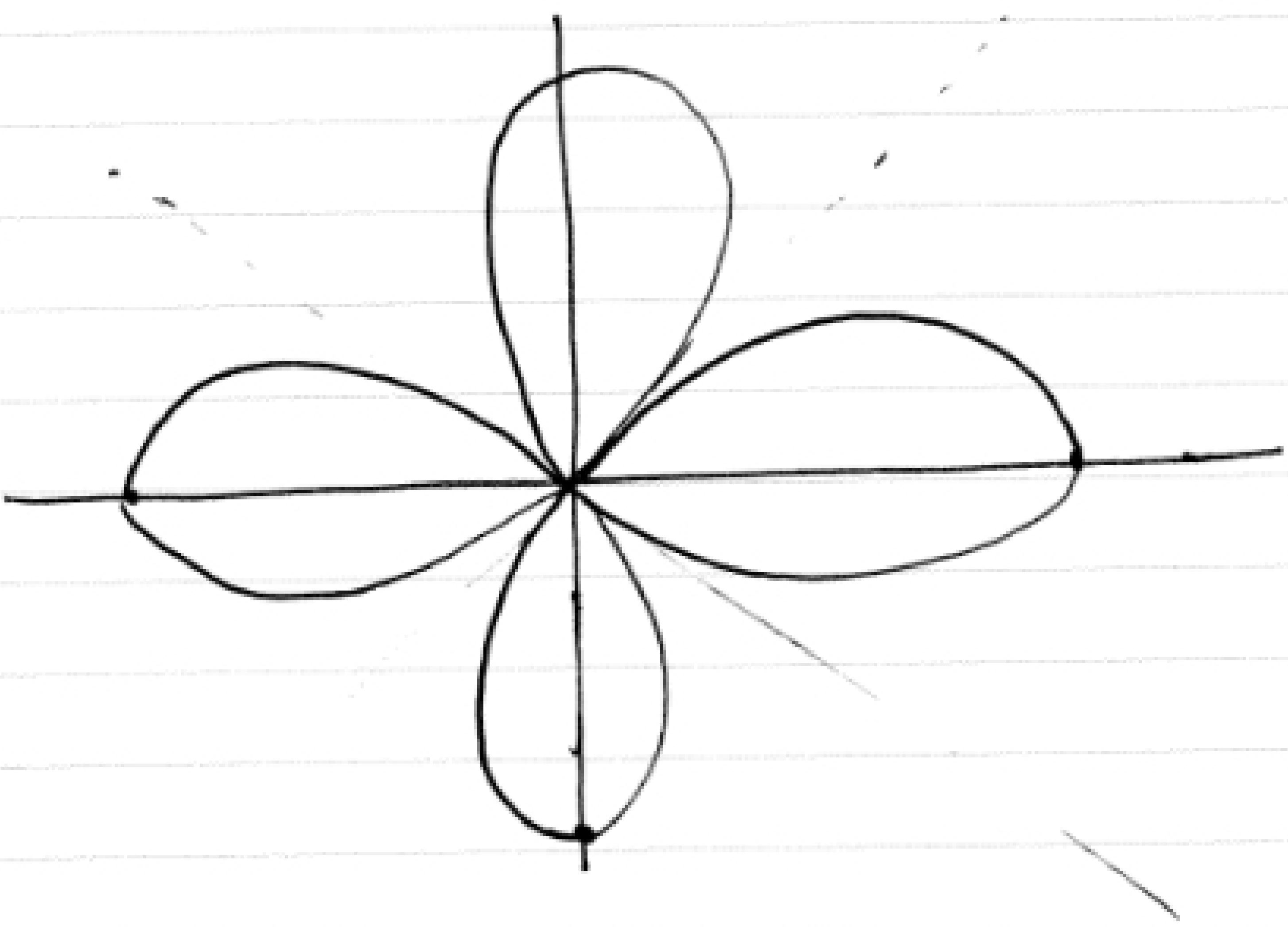
if $\theta \in [0,2\pi]$.

$$\sqrt{x^2+y^2} = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}.$$

$$x^2+y^2 = x$$
$$x^2 - x + y^2 = 0$$
$$x^2 - x + 1/4 + y^2 = 1/4$$
$$(x - 1/2)^2 + y^2 = (1/2)^2$$ circle centred at $(1/2, 0)$ radius $1/2$.

$r = \underline{\cos 2\theta}.$





4-leaf rose.