Frequency-Response Techniques: Chapter 6

In Chapter 6 we will discuss the design of controllers using both root locus and frequency response methods. The next few lectures will review and introduce new frequency response methods.

Sinusoidal Steady-State Analysis

Consider the linear time-invariant system where $G(s)$ is BIBO stable.

$$u(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

If we apply a sinusoidal input

$$u(t) = A\cos(\omega_0 t + \phi),$$

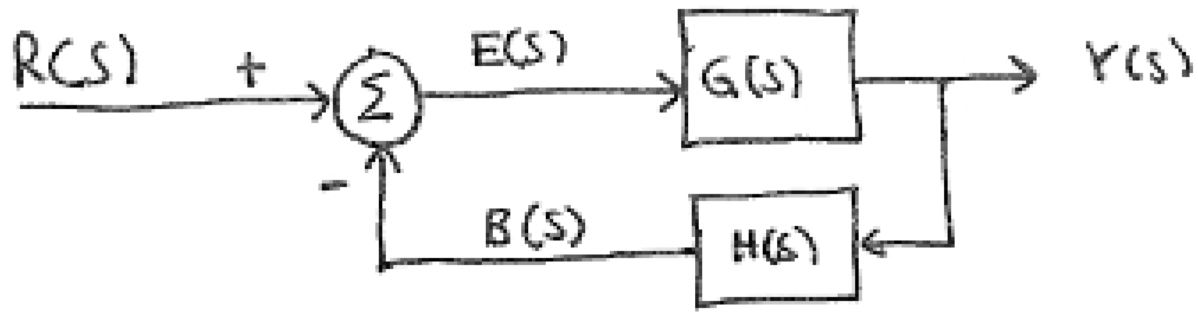
then, following the transient response, the sinusoidal steady-state output is

$$y(t) = A\,|G(j\omega_0)|\cos(\omega_0 t + \phi + \measuredangle G(j\omega_0)),$$

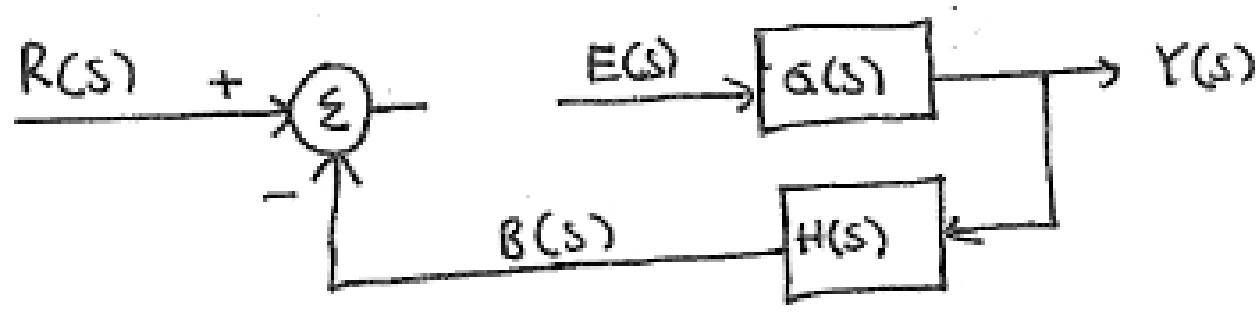
where

$$G(j\omega) = |G(j\omega)|\measuredangle G(j\omega) = G(s)\Big|_{s=j\omega}$$

Consider the closed-loop system



If we break the feedback loop,



the open-loop transfer function is defined as

$$\frac{B(s)}{E(s)} = G(s)H(s)$$

The corresponding frequency response function

$$\frac{B(j\omega)}{E(j\omega)} = G(j\omega)\,H(j\omega)$$

has magnitude

$$M(\omega) = |G(j\omega)H(j\omega)|$$

and phase

$$\phi(\omega) = \measuredangle\, G(j\omega)\,H(j\omega).$$

Using this notation,

$$G(j\omega)\,H(j\omega) = |G(j\omega)\,H(j\omega)| \measuredangle G(j\omega)\,H(j\omega).$$

We will use three different formats to represent the frequency response function $G(j\omega)\,H(j\omega)$

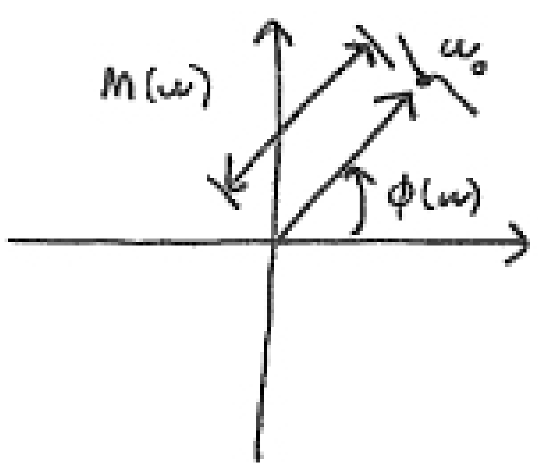
1. <u>Bode Magnitude and Phase Plots</u>

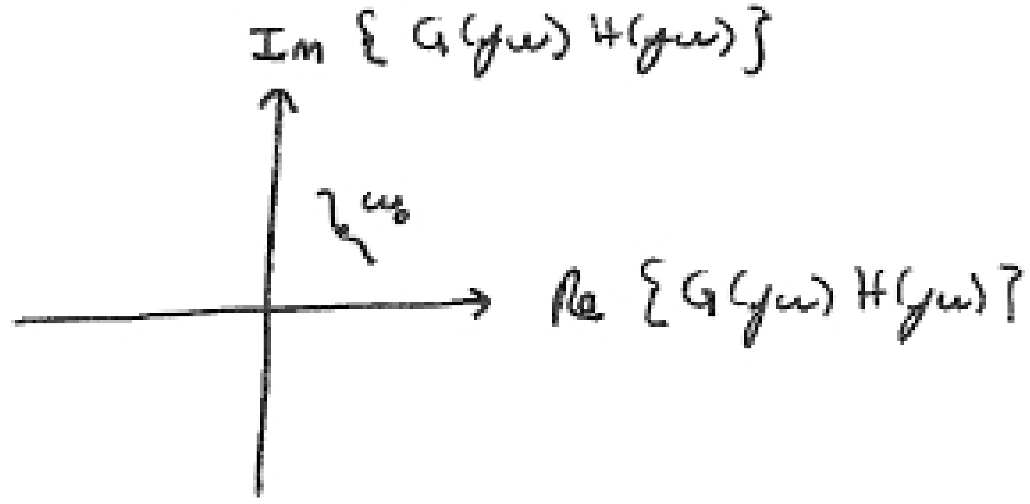
$|G(j\omega)\,H(j\omega)|_{dB}$ versus ω

$\measuredangle G(j\omega)\,H(j\omega)$ versus ω

2. <u>Polar Plot</u>

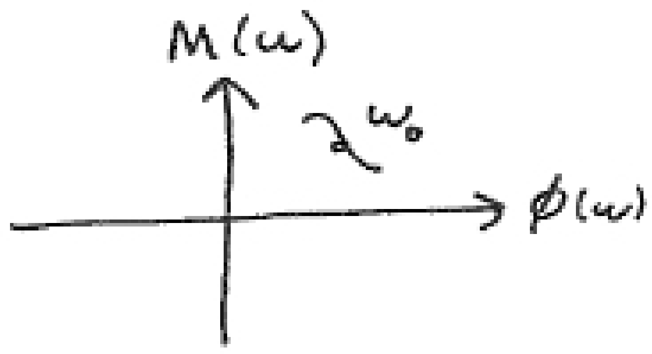
$$G(j\omega)\,H(j\omega) = M(\omega)\,e^{\measuredangle \phi(\omega)}$$





sketch as ω is varied from 0 to infinity.

3. <u>Magnitude versus Phase Plot</u>



sketch as ω varies from 0 to infinity