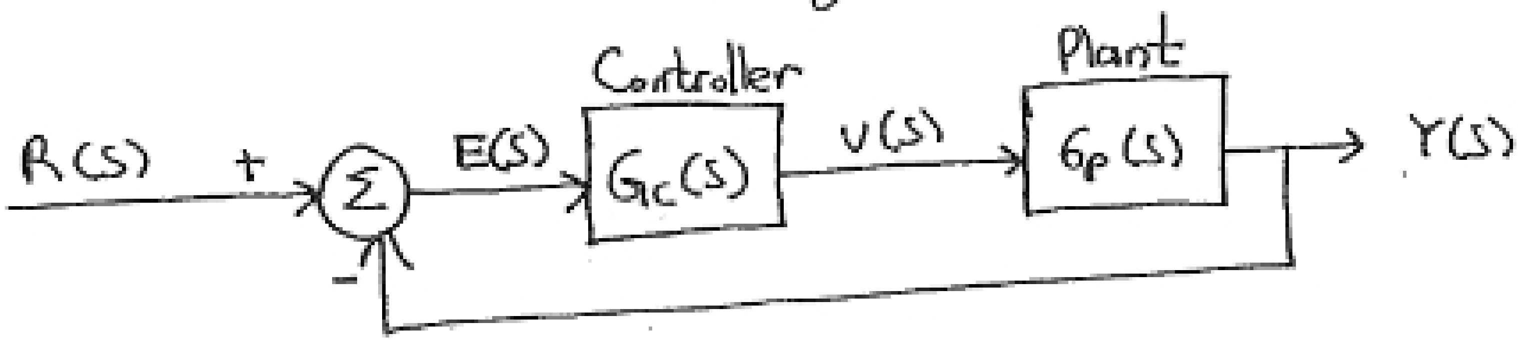
Control System Analysis using Bode Plots:

Determining the relative degree of stability

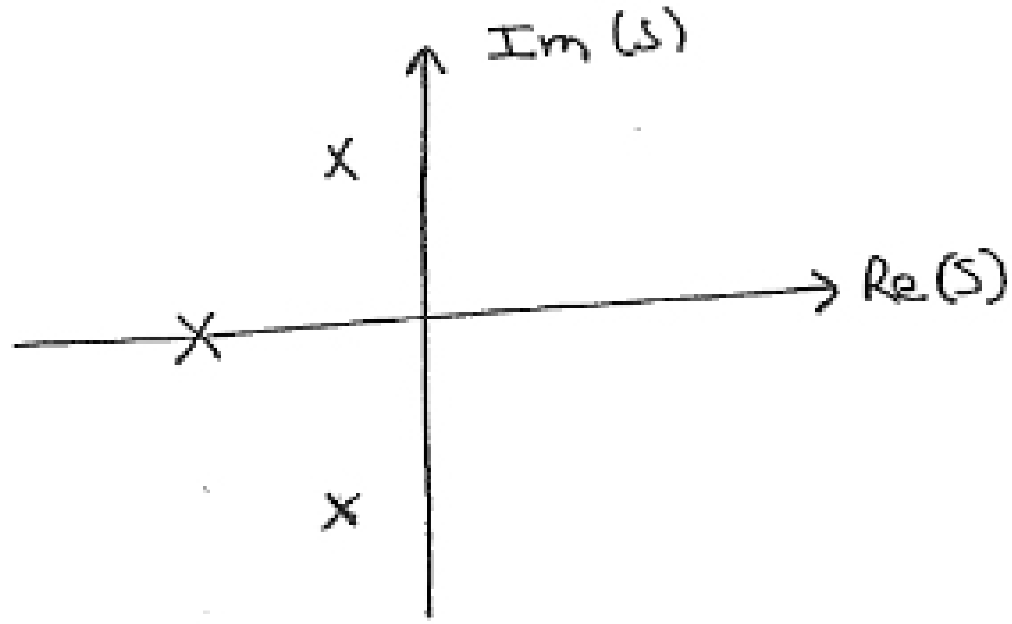
Consider the closed-loop system



where

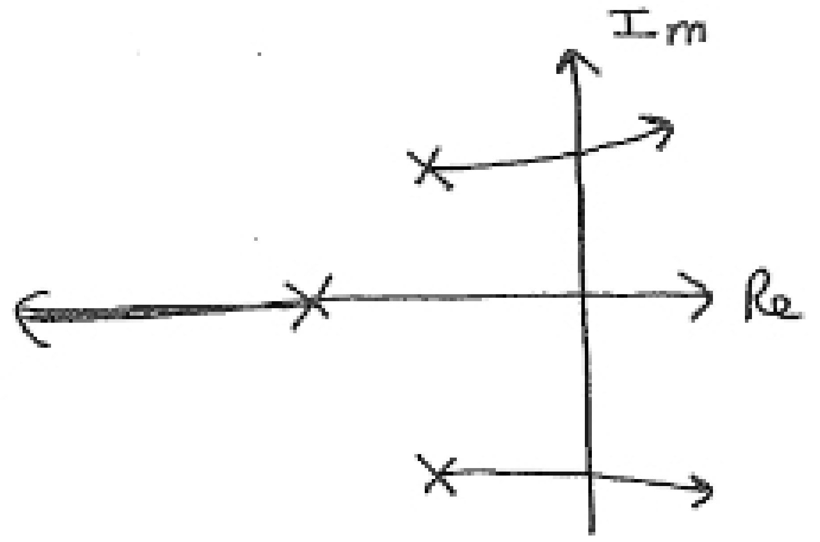
$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}.$$

Suppose that the closed-loop system is stable and has the pole-zero plot



It may be desirable to increase the controller gain, for example, to improve the steady-state accuracy. If we replace $G_c(S)$ by $K_o G_c(S)$, how large can K_o be before the system becomes unstable?

In our example, as K_o increases, the root locus eventually crosses the $j\omega$ axis:



× closed-loop poles $K_o = 1$

— root locus for $K_o > 1$

We can use the Bode magnitude and phase plots to determine the gain K_o at which the poles cross the $j\omega$ axis.

Recall that

$$\frac{Y(j\omega)}{R(j\omega)} = H(s)\Big|_{s=j\omega}.$$

When K_o is chosen so that the poles are on the $j\omega$ axis, it follows that

$$\left|\frac{Y(j\omega)}{R(j\omega)}\right| = \left|\frac{K_o\, G_c(j\omega)\, G_p(j\omega)}{1 + K_o\, G_c(j\omega)\, G_p(j\omega)}\right| = \infty$$

This condition occurs when

$$K_o\, G_c(j\omega)\, G_p(j\omega) = -1$$

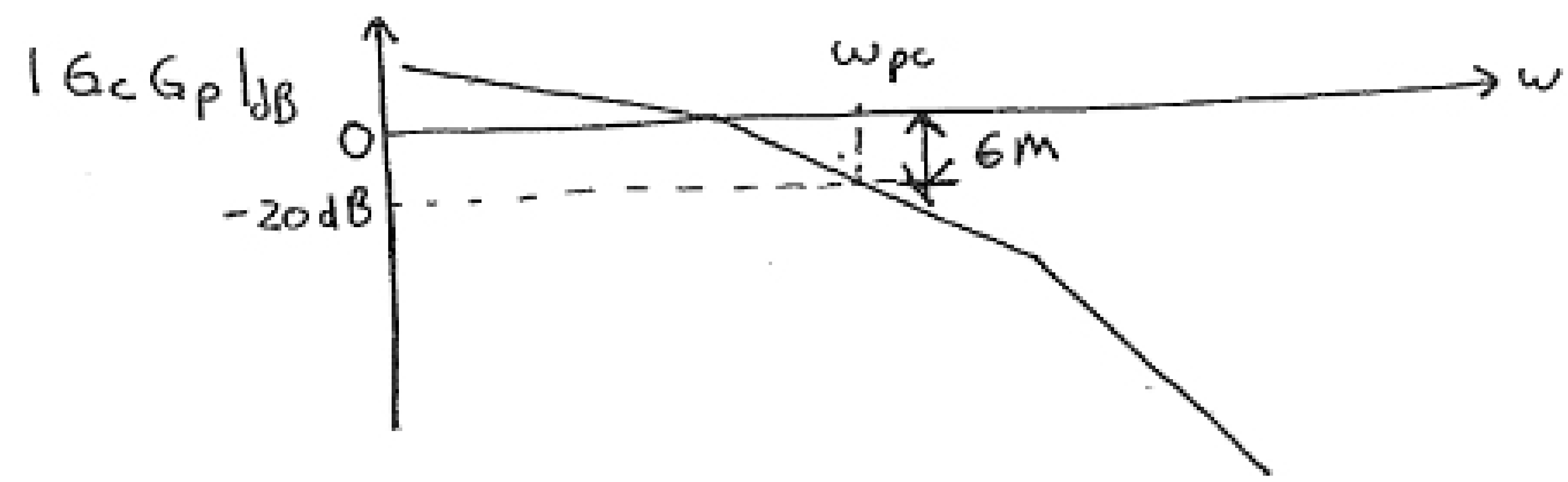
Equivalently,

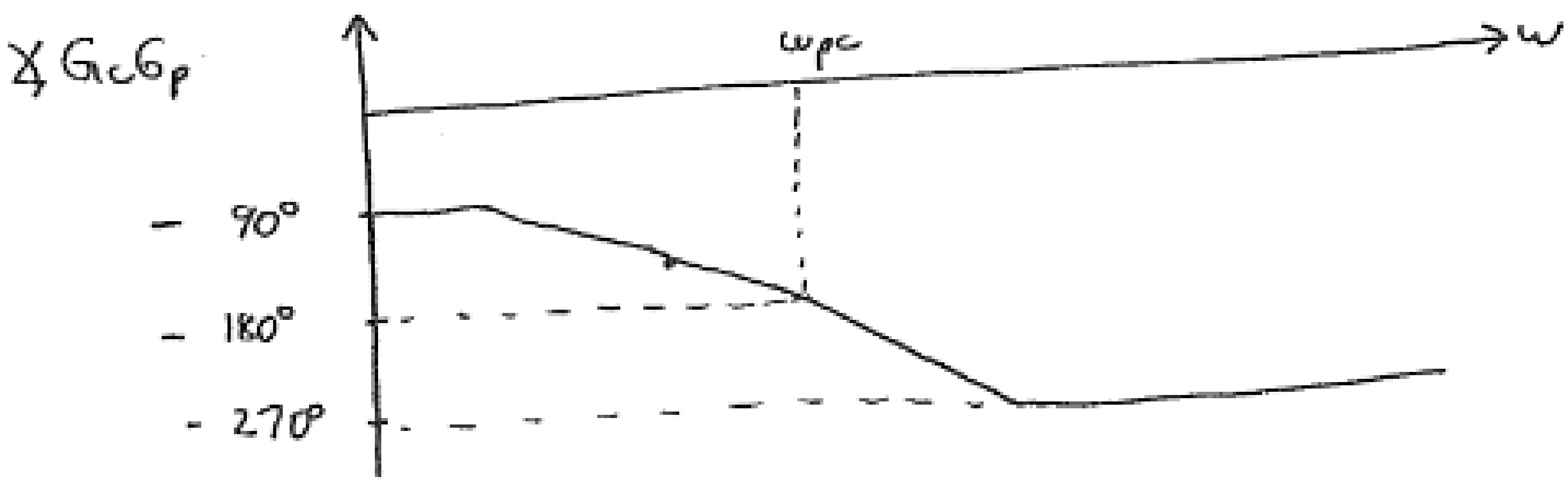
$$|K_o\, G_c(jw)\, G_p(jw)| = 1$$

$$\measuredangle K_o\, G_c(jw)\, G_p(jw) = -180^\circ \pm 360^\circ l$$

Given the Bode magnitude and phase plot of $G_c(jw)\,G_p(jw)$, we can determine the value of K_o that satisfies the above conditions.

<u>Example 1</u>





$\omega_{pc} \equiv$ <u>phase crossover frequency</u>, the frequency at which $\measuredangle G_c(j\omega_{pc})\, G_p(j\omega_{pc}) = -180^\circ$

$GM \equiv$ <u>gain margin</u>, the gain required for $|G_c(j\omega_{pc})\, G_p(j\omega_{pc})| = 1$.