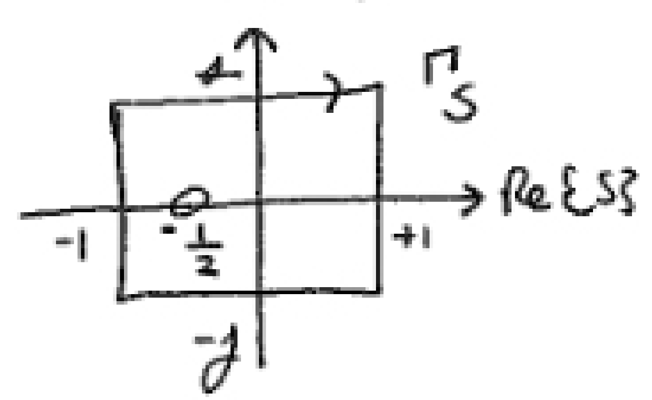
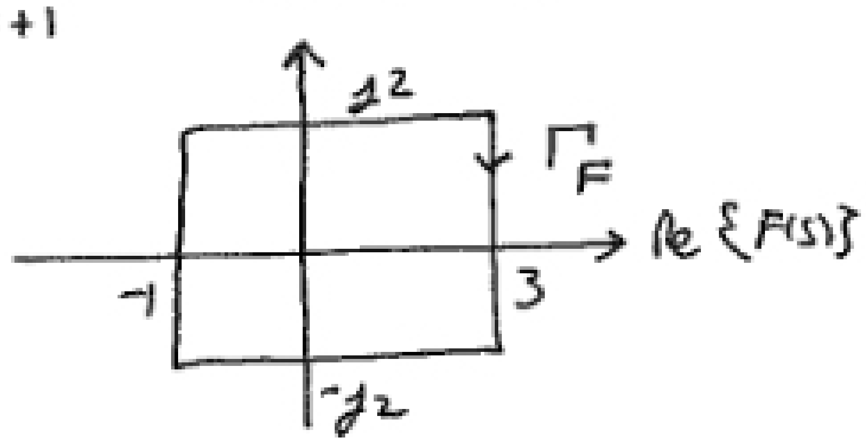
The Nyquist Criterion

In the last lecture we showed how to map a contour Γ_s in the s-plane to a contour Γ_F in the F(S)-plane using the mapping F(S):

$$F(S) = 2S + 1$$



s-plane



F(S) - plane

The encirclement of the poles and zeros of F(S) in the s-plane can be related to the encirclement of the origin in the F(S)-plane by Cauchy's Principle of the Argument theorem.

Cauchy's Theorem (Principle of the Argument)

If a contour Γ_s in the s-plane

1. encircles Z zeros and P poles of $F(s)$,
2. does not pass through any poles or zeros of $F(s)$, and
3. the traversal is in the clockwise direction along the contour Γ_s,

then the contour Γ_F in the $F(s)$-plane encircles the origin of the $F(s)$ plane

$$N = Z - P$$

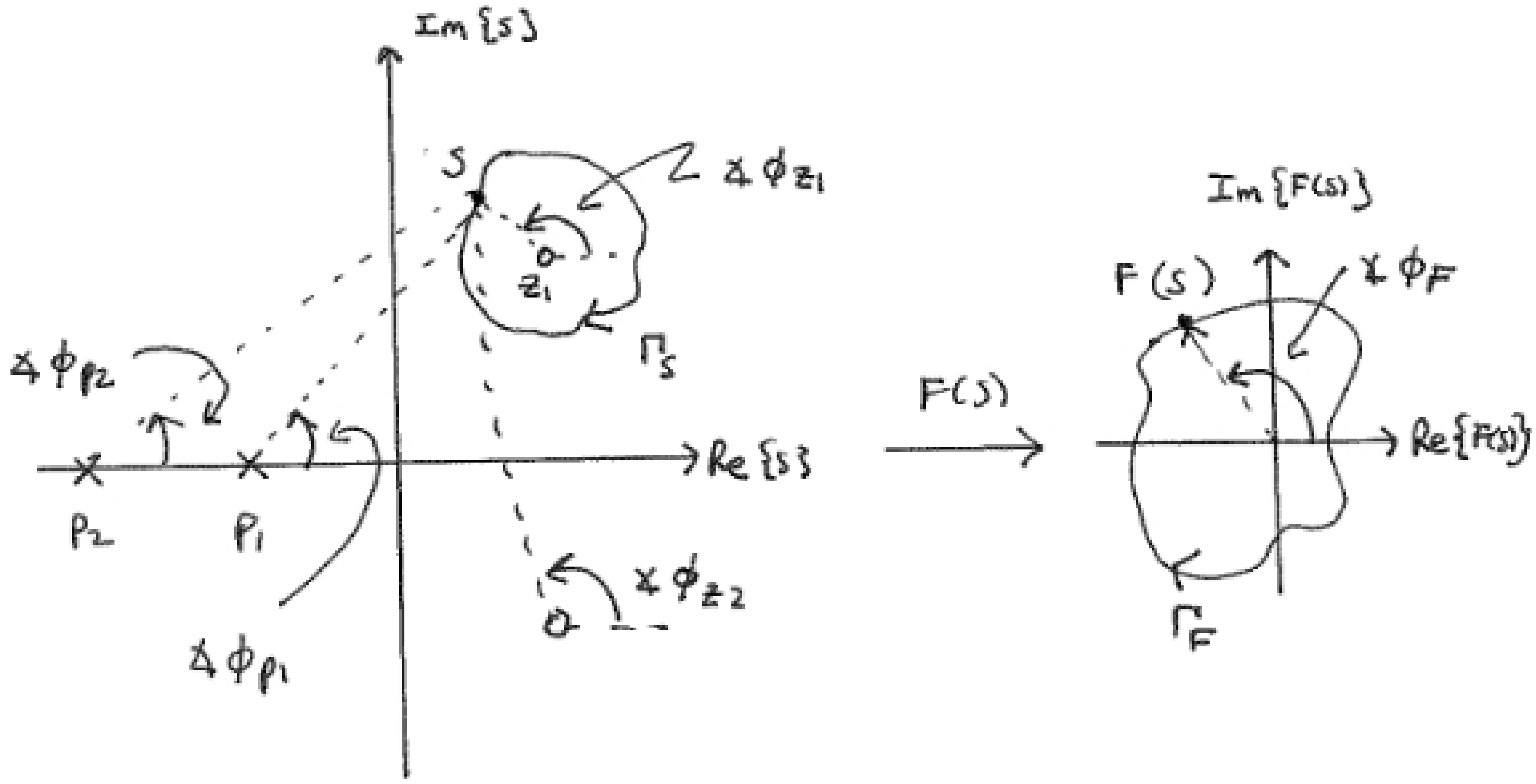
times in the clockwise direction.

Looking at our previous example where $F(s) = 2s + 1$, the contour Γ_s encircles a single zero $(Z = 1,\ P = 0)$. Note that Γ_F encircles the origin of the $F(s)$-plane once $(N = 1)$.

To understand why Cauchy's theorem is true, consider

$$F(s) = \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}$$

and the contour Γ_s



Using this notation,

$$F(s) = |F(s)|\, e^{j\measuredangle\phi_F} = |F(s)|\, e^{j(\measuredangle\phi_{z1} + \measuredangle\phi_{z2} - \measuredangle\phi_{p} - \measuredangle\phi_{p2})}$$

As we move the point s completely around the contour Γ_s, the net change in $\measuredangle\phi_{p1}$, $\measuredangle\phi_{p2}$, $\measuredangle\phi_{z2}$ is zero. On the other hand, $\measuredangle\phi_{z1}$ changes by $+360°$, and so the point $F(s)$ makes one encirclement of the origin in the F-plane in the clockwise direction.

net change in $\measuredangle\phi_F$

$= $ net change in $(\measuredangle\phi_{z1} + \measuredangle\phi_{z2} - \measuredangle\phi_{p1} - \measuredangle\phi_{p2})$.