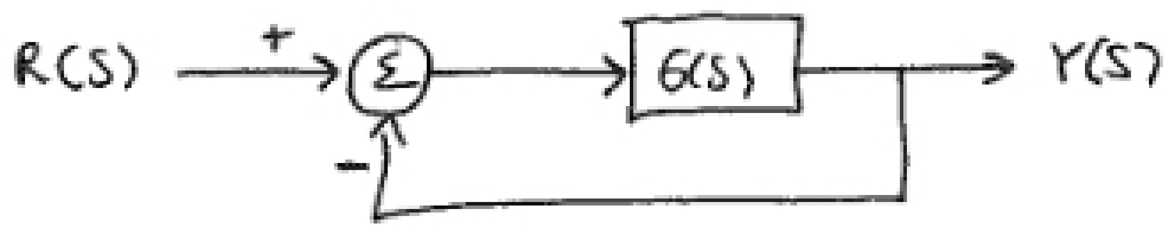
Nyquist Stability Criterion

Consider the closed-loop system



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

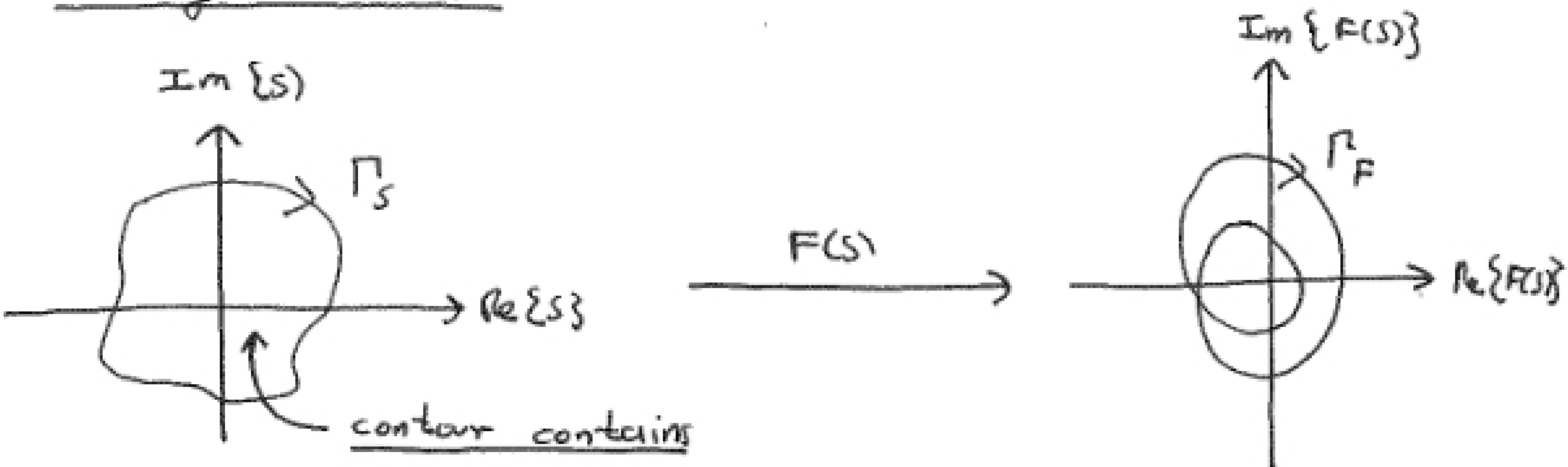
Note the following

① The number of closed-loop poles in the RHP is equal to the number of zeros of $1 + G(s)$ in the RHP.

② The number of poles of $1 + G(s)$ in the RHP is the number of poles of the open-loop transfer function $G(s)$ in the RHP.

In order to determine the number of zeros of $1 + G(s)$ in the RHP, Nyquist used Cauchy's theorem (Principle of the Argument)

Cauchy's Theorem



contour contains
Z zeros of F(S)
P poles of F(S)

$N \equiv$ # clockwise encirclements of origin by Γ_F

$$N = Z - P$$

Comments

① Poles and zeros of F(S) outside the contour Γ_s have no affect on N

② $N > 0 \Rightarrow$ there are more clockwise than counterclockwise encirclements

$N < 0 \Rightarrow$ there are more counterclockwise than clockwise encirclements.

By choosing

① Γ_s to encompass the entire right half plane

② $F(S) = 1 + G(S)$

we have

① Z represents # of closed-loop poles in the RHP

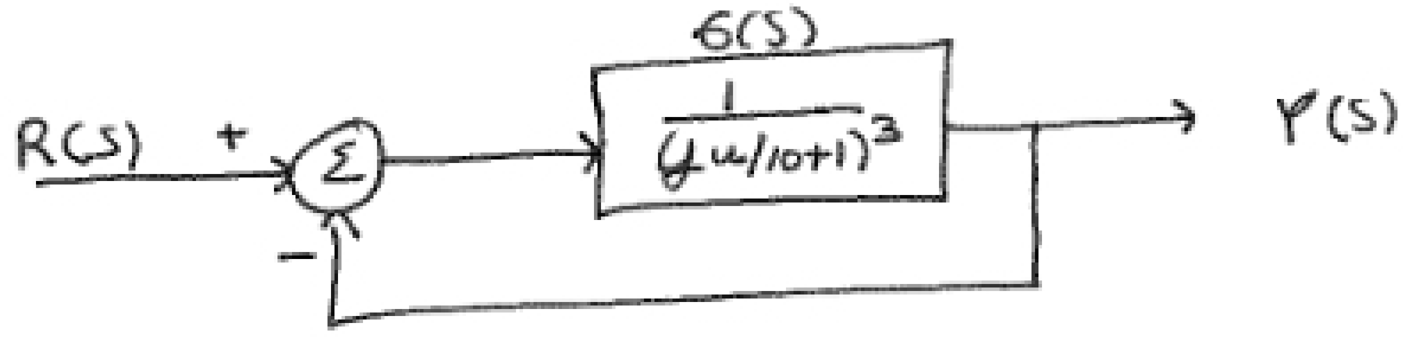
② P represents # of open-loop poles (of G(S)) in the RHP

③ $Z = N + P$.

If we use $F(s) = G(s)$ instead of $F(s) = 1 + G(s)$, then N represents the number of clockwise encirclements of the point $-1 + j0$.

Example 1

Consider the closed-loop system



Is the closed-loop system stable? Apply the Nyquist stability test:

- Draw polar plot of $G(j\omega)$ for $0 < \omega < \infty$
- Complete Nyquist plot by drawing $G(j\omega)$ for $-\infty < \omega < 0$ (using the fact $G(-j\omega) = G^*(j\omega)$).
- Calculate the # of encirclements of the point $-1 + j0$
- Determine

$$Z = \underbrace{N}_{\#\text{ encirclements of } -1+j0} + \overbrace{P}^{\#\text{ poles of } G(s) \text{ in RHP}}$$