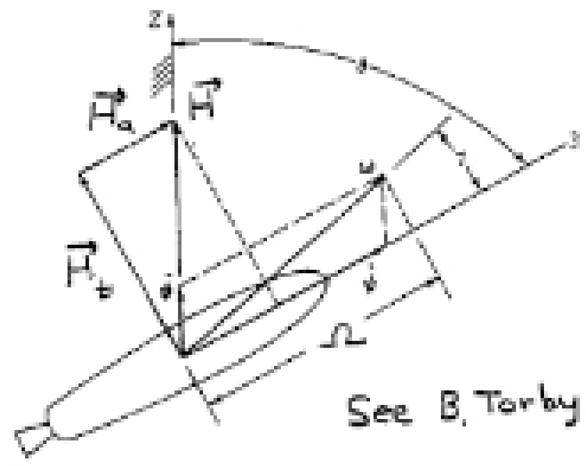
AAE 340
Lecture # 43

Free Motion Symmetric RB Demo



See B. Torby

Following Torby, we have

$$H \cos\theta = I_a \Omega \qquad (7.82)$$

$$\Rightarrow \quad H = I_a \Omega / \cos\theta$$

$$H = I_t \dot{\phi} \qquad (7.84)$$

$$\Rightarrow \quad \frac{I_a \Omega}{\cos\theta} = I_t \dot{\phi}$$

$$\boxed{\dot{\phi} = \frac{I_a \Omega}{I_t \cos\theta}}$$

where $\dot{\phi}$ = precession rate

$\Omega = \omega_g$ = total spin

Note that θ = constant.

If θ is small, then $\cos\theta = 1$ and we have

$$\boxed{\dot{\phi} = \frac{I_a}{I_t} \Omega} \quad \text{for small } \theta$$

Feynman's Wobbling Plate. Explain (***mathematically***) the following excerpt from *Surely You're Joking, Mr. Feynman* by Richard Feynman:

"Within a week I was in the cafeteria and some guy, fooling around, throws a plate in the air. As the plate went up in the air I saw it wobble, and I noticed the red medallion of Cornell on the plate going around. It was pretty obvious to me that the medallion went around faster than the wobbling.

I had nothing to do, so I start to figure out the motion of the rotating plate. I discover that when the angle is very slight, the medallion rotates twice as fast as the wobble rate—two to one. It came out of a complicated equation! Then I thought, 'Is there some way I can see in a more fundamental way, by looking at the forces or the dynamics, why it's two to one?'

I don't remember how I did it, but I ultimately worked out what the motion of the mass particles is, and how all the accelerations balance to make it come out two to one."

Did Feynman remember the story correctly?

Feynman's Wobbling Plate

$$I_a = 2 I_t$$

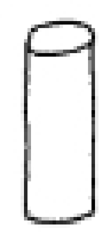
For small θ:

$$\dot{\phi} = \frac{I_a}{I_t} \Omega = \frac{2 I_t}{I_t} \Omega$$

$$\boxed{\dot{\phi} = 2\Omega}$$

$\Rightarrow$ Precesses twice as fast as total spin!

The Long Cylinder Problem



$$\dot{\phi} = \frac{I_a \Omega}{I_t \cos\theta}$$

For small $\theta \Rightarrow \cos\theta \approx 1$

$$\Rightarrow \quad \dot{\phi} = \frac{I_a \Omega}{I_t}$$

Let $\boxed{I_t = n I_a}$

where $n > 1$ for long cylinder

Then $\dot{\phi} = \frac{I_a \Omega}{n I_a}$

$$\Rightarrow \quad \boxed{\dot{\phi} = \frac{\Omega}{n}}$$

If $n = 10 \Rightarrow \dot{\phi} = \frac{1}{10}\Omega$

A spinning pencil precesses very slowly compared to total spin.

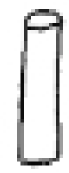
Special Cases

For small θ

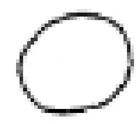
$$\dot{\phi} = \frac{\Omega}{n}$$

where $n = \frac{I_t}{I_a}$

Thin Disk
$n = \frac{1}{2}$



Long Cylinder
e.g. $n = 10$ (pencil)



Sphere
$n = 1$

wobbles faster than spin

Spins faster than wobble

Softball
Spins and wobbles the same
$\Rightarrow$ never wobbles