Chapter 16

11/13/14

$\int_a^b f(x)\,dx$
$\iint f(xy)\,dx\,dy$
$\iiint f(xyz)\,dx\,dy\,dz$
} Chapter 15

Curve $\mathcal{C}$ $\quad \vec{r}: I \longrightarrow \mathbb{R}^3 \quad a \le t \le b$
$t \mapsto \langle x(t), y(t) \rangle$
arc length $= \int_a^b \left|\frac{d\vec{r}}{dt}\right| dt$
} Chapter 13

Generalization
1) More interesting integrands
2) Surfaces
} New!

$\mathbb{R}$: $F'(x) = f(x)$
$\int_a^b f(x)\,dx = F(b) - F(a)$
Generalization is called Stokes Theorem.

$f(xy) = \sqrt{x^2+y^2}$
$C: \vec{r}(t) = \langle 4\cos t, 4\sin t, 3t \rangle \quad -2\pi \le t \le 2\pi$

$\int_C f(s)\,ds$

$= \int_{-2\pi}^{2\pi} f(\vec{r}(t)) \frac{ds}{dt}\,dt$

$\vec{r}\,'(t) = \langle -4\sin t, 4\cos t, 3 \rangle$
$|\vec{r}\,'(t)| = \sqrt{16\sin^2 t + 16\cos^2 t + 9} = 5 = \frac{ds}{dt}$
$f(\vec{r}(t)) = f\langle 4\cos t, 4\sin t, 3t \rangle$
$= \sqrt{16\cos^2 t + 16\sin^2 t} = 4$

$\int_{-2\pi}^{2\pi} 4 \cdot 5\,dt = \int_{-2\pi}^{2\pi} 20\,dt = \boxed{80\pi}$

Wire $\vec{r}(t) = \langle 0, t^2-1, 2t \rangle \quad -1 \le t \le 1$

Density $\delta(xyz) = 15\sqrt{y+2}$

$\bar{X} = \frac{M_x}{m} \quad \bar{Y} = \frac{M_y}{m} \quad \bar{Z} = \frac{M_z}{m}$

$= 0 \quad = -3/5 \quad = 0$

$\vec{r}' = \langle 0, 2t, 2 \rangle$

$\frac{ds}{dt} = |r'(t)| = \sqrt{0+4t^2+4} = 2\sqrt{t^2+1}$

$\delta(x(t), y(t), z(t)) = 15\sqrt{y+2} = 15\sqrt{t^2+1}$

$M = \int_a^b \delta(xyz) \frac{ds}{dt}\, dt$

$\int_{-1}^{1} 15\sqrt{t^2+1} \cdot 2\sqrt{t^2+1}\, dt$

$30\int_{-1}^{1} t^2+1\, dt$

$30(1/3 t^3 + t \Big|_{-1}^{1})$

$30(4/3 - -4/3) = \underline{80} \qquad M = 80$

$M_y = \int_a^b y\,\delta(xyz) \frac{ds}{dt}\, dt$

$= \int_{-1}^{1} 15\sqrt{t^2+1} \cdot 2\sqrt{t^2+1}(t^2-1)\, dt$

$= 30\int (t^2+1)(t^2-1)\, dt$

$30\int (t^4-1)\, dt$

$30(1/5 t^5 - t \Big|_{-1}^{1}) = \underline{-48} \qquad M_y = -48$

$Y = \frac{-48}{80} = \frac{-3}{5}$

$M_z = \int_{-1}^{1} 30(t^2+1)(2t)\, dt$

$60\int 2t^3+2t\, dt$

$60(1/2 t^4 + t^2 \Big|_{-1}^{1})$

$60(3/2 - 3/2) = 0 \qquad M_z = 0$

$\bar{Z} = \frac{0}{80} = 0$

Center of Mass $= \langle 0, -3/5, 0 \rangle$

* on Y-axis