Let's go through a problem that utilizes both of these rules:

Here are the rules for a valid phone number:

1) The first three digits must all be different.
2) The first digit can not be a 1.
3) If the first digit is a 5, the last four digits must all be the same.

Based on our rules, we must group our phone numbers into two separate groups:

A: All numbers that start with 5
B: All numbers that do NOT start with 5

First, we will count all the phone numbers that fit in group A.

___ ___ ___ - ___ ___ ___ ___
1 9 8 10 1 1 1

For the first number, we have 1 choice, 5.
For the second number, we have 9 choices, everything but 5.
For the third number, we have 8 choices.
For the fourth number we have all 10 choices, but for the rest we only have 1 choice, based on our rules.

Thus, we have 1x9x8x10x1x1x1 = 720 phone numbers that start with 5.

Now, let's count the phone numbers that do not start with 5:

___ ___ ___ - ___ ___ ___ ___ ,
8 9 8 10 10 10 10

**for a total of 8x9x8x10x10x10x10 = 5760000 phone numbers that do not start with 5.**

**Using the sum rule, we add this to our other value to get a total of 5,760,720 total possible phone numbers under the given rules.**

**Here is a quick problem for you guys:**

**How many 3 letter words can you form that have two consonants and a vowel?**

# Permutations

**A permutation of objects is simply an arrangement of those objects. A common example of permutations deals with words. How many "words" can you form out of the letters A, B, C, and D, without repeating a letter. Essentially, the question boils down to how many ways can you order a set of objects. In this case, we know we have four choices for the first object. Once we have made that choice, it is clear that we will always have three choices for the next object. In particular, we find the following:**

**____ ____ ____ ____**
**4 x 3 x 2 x 1 = 24 possible permutations.**

**In general, imagine the number of ways to permute n objects. Using the same process as above we find:**

**____ ____ ____ ____**
**n x(n-1)x ... 2 x 1 = n! possible permutations.**

**(Note: For those of you unfamiliar with the ! notation, it is read as "factorial". n! is simply defined as the product of all the positive integers in between 1 and n, inclusive.)**

**Now, consider the following problem:**

**Given n distinct objects, how many permutations are there of r of those objects, where $1 \leq r \leq n$.**

**Using the product rule as before, we find:**

**____ ____ ... ______**
**n x(n-1)x ... (n–k+1) = n!/(n–k)! possible permutations.**