7-8-14

$y'' - 4y = f(t)$

$f(t) = \begin{cases} 2 & 0 < t < 3 \\ 4 & 3 \le t < 4 \\ 0 & t \ge 4 \end{cases}$ $y(0) = 0$ $y'(0) = 1$

$2 + 2u_3 t - 4u_4 t$

$s^2\mathcal{L}_y - sy(0) + y'(0) - 4\mathcal{L}_y = \frac{2}{s} + \frac{2e^{-3t}}{s} - \frac{4e^{-4t}}{s}$

$s^2\mathcal{L}_y + 1 - 4\mathcal{L}_y = \frac{2 + 2e^{3t} - 4e^{4t}}{s}$

$(s^2 - 4)\mathcal{L}_y = (2 + 2e^{3t} - 4e^{4t})\left(\frac{1}{s}\right) + 1$

$\mathcal{L}_y(2 + 2e^{-3t} - 4e^{-4t})\frac{1}{s(s^2-4)} + \frac{1}{s^2-4}$

$\frac{a}{s} + \frac{b}{s-2} + \frac{c}{s+2}$

$as^2 - 4a + bs^2 + cs$

$-4a = 1$

$a = -1/4$

$c = 1/8$

$b = 1/8$

$\frac{a}{s-2} + \frac{b}{s+2}$

$as + 2a$ $bs - b2$

$2a - 2b = 1$

$a + b = 0$

$b = -1/4$

$a = 1/4$

$1/4 e^{2t} - 1/4 e^{-2t}$

$\mathcal{L}_y = (2 + 2e^{3t} - 4e^{4t})(-1/4 + 1/8 e^{2t} + 1/4 e^{-2t})$

$(-1/2 + 1/4 e^{2t} + 1/4 e^{-2t}) + u_3(-1/2 + 1/4 e^{(2t-6)} - 1/4 e^{(-2t+6)}) - u_4(1 + 1/2 e^{(2t-8)} + 1/2 e^{(-2t+8)})$

$+ 1/4 e^{2t} - 1/4 e^{-2t}$

$\frac{e^{t} - e^{-t}}{} = \sinh(t)$

7/9/14

$e^{-\theta} = e^{i(i\theta)} = \cos i\theta + i\sin i\theta$

$e^{\theta} = e^{-i(i\theta)} = \cos(i\theta) - i\sin(i\theta)$

$e^{\theta} + e^{-\theta} = 2\cos(i\theta)$

$\frac{e^{\theta} + e^{-\theta}}{2} = \cos(i\theta) = \cosh(\theta)$

$y'' + 4y = g(t)$ $y(0) = 0$ $y'(0) = 1$ $g(t) \begin{cases} 2 & \text{if } 0 < t < 3 \\ 5 & \text{if } t \ge 3 \end{cases}$

$s^2\mathcal{L}_y - sy(0) - y'(0) + 4\mathcal{L}_y = \mathcal{L}\, 2 + 3u_3$

$\mathcal{L}_y(s^2+4) - 1 = \frac{1}{s} + \frac{3e^{-3s}}{s}$

$\mathcal{L}_y = 1 + 3e^{-3s}\left(\frac{1}{s(s^2+4)}\right) + \frac{1}{s^2+4}$

$\frac{a}{s} + \frac{bs + c}{s^2+4}$

$as^2 + 4a + bs^2 + cs = 1$

$a + b = 0$ $c = 0$

$4a = 1$

$a = 1/4$

$b = -1/4$

$\mathcal{L}_y = 1 + 3e^{-3s}\left(1/4\left(\frac{1}{s}\right) - 1/4\left(\frac{s}{s^2+4}\right)\right) + 1/2 \sin 2t$

$\mathcal{L}_y = 1/4 - 1/4\cos 2t + 3u_3(t)(1/4 - 1/4\cos(2t - 6)) + 1/2\sin 2t$

$y'' + 2y' + 3y = g(t)$ $g(t) \begin{cases} 0 & t < \pi \\ 4 & \text{if } \pi \le t \end{cases}$ $y(0) = 0$ $y'(0) = 0$

$s^2\mathcal{L}_y - \cancel{sy(0)} - \cancel{y'(0)} + 2s\mathcal{L}_y - \cancel{y(0)} + 3\mathcal{L}_y = \mathcal{L}(4u_{\pi}(t))$

$(s^2 + 2s + 3)\mathcal{L}_y = \frac{4e^{-\pi s}}{s}$

$\mathcal{L}_y = 4e^{-\pi s}\left(\frac{1}{s(s^2+2s+3)}\right)$

$\frac{a}{s} + \frac{bs + c}{s^2 + 2s + 3}$

$as^2 + 2sa + 3a + bs^2 + cs = 1$

$a + b = 0$

$2a + c = 0$

$3a = 1$

$a = 1/3$

$c = -2/3$

$b = -1/3$

$\mathcal{L}_y = 4e^{-\pi s}\left(1/3\left(\frac{1}{s}\right) + \frac{(-1/3)s - 2/3}{s^2 + 2s + 3}\right)$

$(s^2 + 2s + 1) + 2$

$\frac{-1/3 s - 2/3}{(s+1)^2 + 2} = -1/3\left(\frac{(s+1) + 1}{(s+1)^2 + 2}\right)$

$e^{-t}\cos\sqrt{2}t + e^{-t}\sin\sqrt{2}t$

$\left(\frac{s+1}{(s+1)^2 + 2}\right.$ $\left.\frac{1}{\sqrt{2}}\frac{\sqrt{2}}{(s+1)^2 + 2}\right)$

$\mathcal{L} = -1/3 \cdot 4 \cdot e^{-\pi s}(-1 + e^{-t}\cos\sqrt{2}t + e^{-t}\sin\sqrt{2}t)$

$-4/3\, u_{\pi}\left(-1 + e^{-t+\pi}\cos\sqrt{2}(t-\pi) + \frac{1}{\sqrt{2}}e^{-t+\pi}\sin(\sqrt{2}(t-\pi))\right)$