29. We have $\lim\limits_{x\to 4}(4x-9)=4(4)-9=7$ and $\lim\limits_{x\to 4}(x^2-4x+7)=4^2-4(4)+7=7$. Since $4x-9\le f(x)\le x^2-4x+7$ for $x\ge 0$, $\lim\limits_{x\to 4} f(x)=7$ by the Squeeze Theorem.

31. $-1\le\cos(2/x)\le 1 \quad\Rightarrow\quad -x^4\le x^4\cos(2/x)\le x^4$. Since $\lim\limits_{x\to 0}(-x^4)=0$ and $\lim\limits_{x\to 0}x^4=0$, we have $\lim\limits_{x\to 0}\left[x^4\cos(2/x)\right]=0$ by the Squeeze Theorem.

32. $-1\le\sin(\pi/x)\le 1 \quad\Rightarrow\quad e^{-1}\le e^{\sin(\pi/x)}\le e^1 \quad\Rightarrow\quad \sqrt{x}/e\le\sqrt{x}\,e^{\sin(\pi/x)}\le\sqrt{x}\,e$. Since $\lim\limits_{x\to 0^+}(\sqrt{x}/e)=0$ and $\lim\limits_{x\to 0^+}(\sqrt{x}\,e)=0$, we have $\lim\limits_{x\to 0^+}\left[\sqrt{x}\,e^{\sin(\pi/x)}\right]=0$ by the Squeeze Theorem.

40. Suppose that $f(3)<6$. By the Intermediate Value Theorem applied to the continuous function f on the closed interval $[2,3]$, the fact that $f(2)=8>6$ and $f(3)<6$ implies that there is a number c in $(2,3)$ such that $f(c)=6$. This contradicts the fact that the only solutions of the equation $f(x)=6$ are $x=1$ and $x=4$. Hence, our supposition that $f(3)<6$ was incorrect. It follows that $f(3)\ge 6$. But $f(3)\ne 6$ because the only solutions of $f(x)=6$ are $x=1$ and $x=4$. Therefore, $f(3)>6$.

44. The equation $\sin x=x^2-x$ is equivalent to the equation $\sin x-x^2+x=0$. $f(x)=\sin x-x^2+x$ is continuous on the interval $[1,2]$, $f(1)=\sin 1\approx 0.84$, and $f(2)=\sin 2-2\approx -1.09$. Since $\sin 1>0>\sin 2-2$, there is a number c in $(1,2)$ such that $f(c)=0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\sin x-x^2+x=0$, or $\sin x=x^2-x$, in the interval $(1,2)$.

45. (a) $f(x)=\cos x-x^3$ is continuous on the interval $[0,1]$, $f(0)=1>0$, and $f(1)=\cos 1-1\approx -0.46<0$. Since $1>0>-0.46$, there is a number c in $(0,1)$ such that $f(c)=0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\cos x-x^3=0$, or $\cos x=x^3$, in the interval $(0,1)$.

(b) $f(0.86)\approx 0.016>0$ and $f(0.87)\approx -0.014<0$, so there is a root between 0.86 and 0.87, that is, in the interval $(0.86,0.87)$.

46. (a) $f(x)=\ln x-3+2x$ is continuous on the interval $[1,2]$, $f(1)=-1<0$, and $f(2)=\ln 2+1\approx 1.7>0$. Since $-1<0<1.7$, there is a number c in $(1,2)$ such that $f(c)=0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\ln x-3+2x=0$, or $\ln x=3-2x$, in the interval $(1,2)$.

(b) $f(1.34)\approx -0.03<0$ and $f(1.35)\approx 0.0001>0$, so there is a root between 1.34 and 1.35, that is, in the interval $(1.34,1.35)$.

55. Define $u(t)$ to be the monk's distance from the monastery, as a function of time, on the first day, and define $d(t)$ to be his distance from the monastery, as a function of time, on the second day. Let D be the distance from the monastery to the top of the mountain. From the given information we know that $u(0) = 0$, $u(12) = D$, $d(0) = D$ and $d(12) = 0$. Now consider the function $u - d$, which is clearly continuous. We calculate that $(u - d)(0) = -D$ and $(u - d)(12) = D$. So by the Intermediate Value Theorem, there must be some time t_0 between 0 and 12 such that $(u - d)(t_0) = 0 \quad \Leftrightarrow \quad u(t_0) = d(t_0)$. So at time t_0 after 7:00 AM, the monk will be at the same place on both days.

14. (a) Let $H(t) = 10t - 1.86t^2$.

$$\begin{aligned} v(1) &= \lim_{h \to 0} \frac{H(1+h) - H(1)}{h} = \lim_{h \to 0} \frac{\left[10(1+h) - 1.86(1+h)^2\right] - (10 - 1.86)}{h} \\ &= \lim_{h \to 0} \frac{10 + 10h - 1.86(1 + 2h + h^2) - 10 + 1.86}{h} \\ &= \lim_{h \to 0} \frac{10 + 10h - 1.86 - 3.72h - 1.86h^2 - 10 + 1.86}{h} \\ &= \lim_{h \to 0} \frac{6.28h - 1.86h^2}{h} = \lim_{h \to 0} (6.28 - 1.86h) = 6.28 \end{aligned}$$

The velocity of the rock after one second is 6.28 m/s.

(b) $$\begin{aligned} v(a) &= \lim_{h \to 0} \frac{H(a+h) - H(a)}{h} = \lim_{h \to 0} \frac{\left[10(a+h) - 1.86(a+h)^2\right] - (10a - 1.86a^2)}{h} \\ &= \lim_{h \to 0} \frac{10a + 10h - 1.86(a^2 + 2ah + h^2) - 10a + 1.86a^2}{h} \\ &= \lim_{h \to 0} \frac{10a + 10h - 1.86a^2 - 3.72ah - 1.86h^2 - 10a + 1.86a^2}{h} = \lim_{h \to 0} \frac{10h - 3.72ah - 1.86h^2}{h} \\ &= \lim_{h \to 0} \frac{h(10 - 3.72a - 1.86h)}{h} = \lim_{h \to 0} (10 - 3.72a - 1.86h) = 10 - 3.72a \end{aligned}$$

The velocity of the rock when $t = a$ is $(10 - 3.72a)$ m/s.

(c) The rock will hit the surface when $H = 0 \quad \Leftrightarrow \quad 10t - 1.86t^2 = 0 \quad \Leftrightarrow \quad t(10 - 1.86t) = 0 \quad \Leftrightarrow \quad t = 0$ or $1.86t = 10$. The rock hits the surface when $t = 10/1.86 \approx 5.4$ s.

(d) The velocity of the rock when it hits the surface is $v\left(\frac{10}{1.86}\right) = 10 - 3.72\left(\frac{10}{1.86}\right) = 10 - 20 = -10$ m/s.

20. Since $(4, 3)$ is on $y = f(x)$, $f(4) = 3$. The slope of the tangent line between $(0, 2)$ and $(4, 3)$ is $\frac{1}{4}$, so $f'(4) = \frac{1}{4}$.

34. By (4), $\displaystyle\lim_{h \to 0} \frac{\sqrt[4]{16+h} - 2}{h} = f'(16)$, where $f(x) = \sqrt[4]{x}$ and $a = 16$.

Or: By (4), $\displaystyle\lim_{h \to 0} \frac{\sqrt[4]{16+h} - 2}{h} = f'(0)$, where $f(x) = \sqrt[4]{16 + x}$ and $a = 0$.

36. By Equation 5, $\displaystyle\lim_{x \to \pi/4} \frac{\tan x - 1}{x - \pi/4} = f'(\pi/4)$, where $f(x) = \tan x$ and $a = \pi/4$.

4. $f(x) = \sqrt{30}$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.

6. $F(x) = \frac{3}{4}x^8 \;\Rightarrow\; F'(x) = \frac{3}{4}(8x^7) = 6x^7$

8. $f(t) = \frac{1}{2}t^6 - 3t^4 + t \;\Rightarrow\; f'(t) = \frac{1}{2}(6t^5) - 3(4t^3) + 1 = 3t^5 - 12t^3 + 1$

16. $y = \sqrt{x}\,(x-1) = x^{3/2} - x^{1/2} \;\Rightarrow\; y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} = \frac{1}{2}x^{-1/2}(3x-1)$ [factor out $\frac{1}{2}x^{-1/2}$]

or $y' = \dfrac{3x-1}{2\sqrt{x}}$.

20. $g(u) = \sqrt{2}\,u + \sqrt{3u} = \sqrt{2}\,u + \sqrt{3}\,\sqrt{u} \;\Rightarrow\; g'(u) = \sqrt{2}\,(1) + \sqrt{3}\left(\frac{1}{2}u^{-1/2}\right) = \sqrt{2} + \frac{\sqrt{3}}{2\sqrt{u}}$

4. By the Product Rule, $g(x) = \sqrt{x}\,e^x = x^{1/2}e^x \;\Rightarrow\; g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x+1)$.

8. $f(t) = \dfrac{2t}{4+t^2} \;\overset{\text{QR}}{\Rightarrow}\; f'(t) = \dfrac{(4+t^2)(2) - (2t)(2t)}{(4+t^2)^2} = \dfrac{8+2t^2-4t^2}{(4+t^2)^2} = \dfrac{8-2t^2}{(4+t^2)^2}$

14. $y = \dfrac{t}{(t-1)^2} = \dfrac{t}{t^2-2t+1} \;\overset{\text{QR}}{\Rightarrow}$

$y' = \dfrac{(t^2-2t+1)(1) - t(2t-2)}{[(t-1)^2]^2} = \dfrac{(t-1)^2 - 2t(t-1)}{(t-1)^4} = \dfrac{(t-1)[(t-1)-2t]}{(t-1)^4} = \dfrac{-t-1}{(t-1)^3}$

18. $z = w^{3/2}(w + ce^w) = w^{5/2} + cw^{3/2}e^w \;\Rightarrow\; z' = \frac{5}{2}w^{3/2} + c\left(w^{3/2}\cdot e^w + e^w\cdot\frac{3}{2}w^{1/2}\right) = \frac{5}{2}w^{3/2} + \frac{1}{2}cw^{1/2}e^w(2w+3)$

2. Let $u = g(x) = 2x^3 + 5$ and $y = f(u) = u^4$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (4u^3)(6x^2) = 24x^2(2x^3+5)^3$.

4. Let $u = g(x) = \cot x$ and $y = f(u) = \sin u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\cos u)(-\csc^2 x) = -\cos(\cot x)\csc^2 x$.

10. $f(x) = (1+x^4)^{2/3} \;\Rightarrow\; f'(x) = \frac{2}{3}(1+x^4)^{-1/3}(4x^3) = \dfrac{8x^3}{3\sqrt[3]{1+x^4}}$