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Homework Review

HW9) $J_1\ddot{\theta}_1 + c(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = k_2 I$

$J_2\ddot{\theta}_2 - c(\dot{\theta}_1 - \dot{\theta}_2) - k(\theta_1 - \theta_2) = 0$

$\mathcal{L} \rightarrow J_1 s^2 \phi_1 + cs\phi_1 - cs\phi_2 + k\phi_1 - \phi_2 = k_2 I(s)$

$\mathcal{L} \rightarrow J_2 s^2 \phi_2 - cs\phi_1 + cs\phi_2 - k\phi_1 + k\phi_2 = 0$

$(J_2 s^2 + cs + k)\phi_2 = (cs+k)\phi_1 \rightarrow \phi_1 = \frac{(J_2 s^2 + cs + k)\phi_2}{cs+k}$

$\rightarrow \left[(J_1 s^2 + cs + k)\left(\frac{J_2 s^2 + cs + k}{cs+k}\right) - (cs+k)\right]\phi_2 = k_2 I(s)$

$\left[(J_1 s^2 + cs + k)(J_2 s^2 + cs + k) - (cs+k)^2\right]\phi_2 = k_2(cs+k) I(s)$

$$\boxed{G(s) = \frac{\phi_2(s)}{I(s)} = \frac{k_2(cs+k)}{(J_1 s^2 + cs + k)(J_2 s^2 + cs + k) - (cs+k)^2}}$$

HW10) $x = \begin{bmatrix}\theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2\end{bmatrix} \quad U = I \quad y = \theta_2$

$\ddot{\theta}_1 = -\frac{k(\theta_1 - \theta_2)}{J_1} - \frac{c(\dot{\theta}_1 - \dot{\theta}_2)}{J_1} + \frac{k_2 I}{J_1}$

$\ddot{\theta}_2 = \frac{k(\theta_1 - \theta_2)}{J_2} + \frac{c(\dot{\theta}_1 - \dot{\theta}_2)}{J_2}$

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k}{J_1} & \frac{k}{J_1} & \frac{-c}{J_1} & \frac{c}{J_1} \\ \frac{k}{J_2} & \frac{-k}{J_2} & \frac{c}{J_2} & \frac{-c}{J_2} \end{bmatrix} \begin{bmatrix}\theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2\end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{k_2}{J_1} \\ 0 \end{bmatrix} U \qquad y = [0\,1\,0\,0]\,x$$

HW11) $y_{ss} = (-CA^{-1}B + D)U \rightarrow$ to big of matrix, use MATLAB

$(cs+k)(cs+k)$

$\rightarrow I = \frac{1}{s}$

$$\theta_{2ss} = \lim_{s\to 0} \cancel{s}\frac{1}{\cancel{s}} \frac{k_2(cs+k)}{(J_1 s^2 + cs + k)(J_2 s^2 + cs + k) - (cs+k)^2}$$

$$\theta_{2ss} = \frac{k_2 k}{k^2 - k^2} = \frac{k_2 k}{0}$$

$$\rightarrow \frac{\dot{\theta}_2(s)}{I(s)} = \frac{s\, k_2(cs+k)}{(J_1 s^2 + cs + k)(J_2 s^2 + cs + k) - (cs+k)^2} = \frac{0}{0}$$

$Y_{ss} = (-CA^{-1}B + D)U$

HW12 $A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{\begin{bmatrix} -\frac{b}{m} & -1 \\ \frac{k}{m} & 0 \end{bmatrix}}{\frac{-b}{m}(0) - (-1)(\frac{k}{m})} \rightarrow -CA^{-1} \dfrac{\begin{bmatrix} \frac{b}{m} & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}}{\frac{k}{m}}$

$\rightarrow -CA^{-1}B = \dfrac{\begin{bmatrix} \frac{b}{m} & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}}{\frac{k}{m}} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} = \dfrac{\begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix}}{\frac{k}{m}} \rightarrow \begin{bmatrix} \frac{1}{k} \\ 0 \end{bmatrix}$

HW13 $\frac{dh}{dt} = \frac{Q_{in}}{A} - \frac{ch}{A}$ $G(s) = \frac{H(s)}{Q_{in}(s)} = \frac{\frac{1}{A}}{s + \frac{c}{A}}$

pole(s) $\rightarrow s + \frac{c}{A} = 0 \rightarrow s = \frac{-c}{A}$

root $\rightarrow h = ae^{\lambda t} \rightarrow \frac{dh}{dt} = \lambda a e^{\lambda t}$

$\lambda a e^{\lambda t} + \frac{cae^{\lambda t}}{A} = 0 \rightarrow \lambda + \frac{c}{A} = 0 \rightarrow \lambda = \frac{-c}{A}$

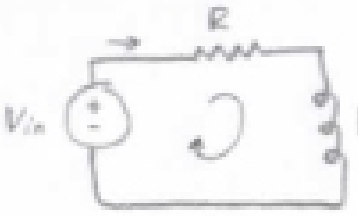
Steady state $\rightarrow ch = 1 \rightarrow h = \frac{1}{c}$

h_{ss} when $Q_{in} = t$ $\mathcal{L}[t] = \frac{1}{s^2}$

$\lim_{s\to 0} s \frac{1}{s^2} \frac{\frac{1}{A}}{s + \frac{c}{A}} \rightarrow \frac{\frac{1}{A}}{s^2 + \frac{cs}{A}} \rightarrow \frac{\frac{1}{A}}{0} \approx \infty$

or

$\cancel{\dot{h}}^{0} = \frac{Q_{in}}{A} - \frac{ch}{A} \rightarrow \frac{t}{A} = \frac{ch}{A} \rightarrow h = \frac{t}{c}$



$-V_{in} + Ri + L\frac{di}{dt} = 0$

$L\frac{di}{dt} + Ri = V_{in}$

$* \ \frac{1}{\text{pole}} = \text{time constant}$

<u>HW14</u> $L\frac{di}{dt} + Ri = V_{in} \longrightarrow \mathcal{L} \rightarrow (Ls + R)\,I(s) = V_{in}(s)$

$G(s) = \frac{I(s)}{V_{in}(s)} = \frac{1}{Ls+R}$

pole $\rightarrow Ls + R = 0 \rightarrow s = \frac{-R}{L} = \frac{1}{T} \longrightarrow$ so time constant, T $\rightarrow T = \frac{-L}{R}$

root $\rightarrow L\lambda + R = 0 \rightarrow \lambda = \frac{-R}{L}$

steady state response $\rightarrow \cancel{L\frac{di}{dt}}^{\,0} + Ri_{ss} = 1 \rightarrow i_{ss} = \frac{1}{R}$

<u>HW15</u> a) $P_{1,2} = -1 \pm 2j \longrightarrow \omega_{n_{1,2}} = \sqrt{1^2+2^2} = \sqrt{5} \rightarrow \zeta_{1,2} = \frac{1}{\sqrt{5}}$

$P_{3,4} = -1 \pm j \longrightarrow \omega_{3,4} = \sqrt{1^2+1^2} = \sqrt{2} \longrightarrow \zeta_{3,4} = \frac{1}{\sqrt{2}}$

b) $P_{1,2} = -1 \pm 2j \rightarrow \omega_{n_{1,2}} = \sqrt{1^2+2^2} = \sqrt{5} \rightarrow \zeta_{12} = \frac{1}{\sqrt{5}}$

$P_{3,4} = -2 \pm 2j \rightarrow \omega_{n_{3,4}} = \sqrt{2^2+2^2} = \sqrt{8} \rightarrow \zeta_{34} = \frac{2}{\sqrt{8}}$

c) $P_{12} = -1 \pm 2j \rightarrow \omega_{n_{12}} = \sqrt{1^2+2^2} = \sqrt{5} \rightarrow \zeta_{12} = \frac{1}{\sqrt{5}}$

$P_{34} = -2 \pm 4j \rightarrow \omega_{34} = \sqrt{2^2+4^2} = \sqrt{20} \rightarrow \zeta_{34} = \frac{2}{\sqrt{20}}$

$\frac{X(s)}{U(s)} = G(s) = \frac{1}{as+b} \qquad a\dot{x} + bx = U \qquad U = 1 \qquad x_p = ?$

$X(s) = U(s)G(s) = \frac{1}{s}\frac{1}{as+b}$

$x(t) = \mathcal{L}^{-1}\left[\frac{1}{s}\frac{1}{as+b}\right] = \mathcal{L}^{-1}\left[\frac{\frac{1}{a}}{s(s+\frac{b}{a})}\right] \overset{(14)}{\Rightarrow} \frac{1}{a}\left(\frac{1}{\frac{b}{a}}\left(1-e^{-\frac{b}{a}t}\right)\right) = \frac{1}{b}\left(1-e^{-\frac{b}{a}t}\right)$

$a\dot{x} + bx = U \longrightarrow x_h = x_0 e^{-\frac{b}{a}t} \qquad x_p = c(t)e^{-b/a\,t} \rightarrow \dot{x}_p = \dot{c}\,e^{-(b/a)t} - \frac{b}{a}c(t)e^{-\frac{b}{a}t}$

$a\dot{c}(t)e^{-\frac{b}{a}t} - bc(t)e^{-\frac{b}{a}t} + bc(t)e^{-\frac{b}{a}t} = 1$

$\dot{c}(t) = \frac{1}{a}e^{(b/a)t} \longrightarrow \frac{dc}{dt} = \int \frac{1}{a}e^{(b/a)t}\,dt \rightarrow c(t) = \frac{1}{b}\left(e^{b/a\,t} - 1\right)$

$x_p = c(t)e^{-\frac{b}{a}t} = \frac{1}{b}\left(e^{\frac{b}{a}t} - 1\right)e^{-b/a\,t}$

$x_p = \frac{1}{b}\left(1 - e^{-\frac{b}{a}t}\right)$