

Functions

- In algebra and calculus, functions are the principal objects of study.
- You probably have seen them in a form like “ $f(x) = x^2 - 3x + 5$ ”.
- This equation defines a function f with independent variable x . It specifies an association between values of x and values of $f(x)$.
- Functions are also common in programming languages;

Examples of functions

- The sets A and B can be any sets whatsoever. So if $P = \{fred, john, mary\}$ and $G = \{A, B, C, D, E\}$ we can let $grade(fred) = D$, $grade(john) = D$, and $grade(Mary) = A$. Then $grade : P \rightarrow G$.
- If A is infinite, we cannot list out the function values this way, so typically we have some kind of generic rule, or a computation method, to specify the values of a function. Thus $square : \mathbb{R} \rightarrow \mathbb{R}$ is given by

$$square(x) = x^2.$$

- Another important type of numerical function is a *linear* function:

$$L(x) = 3x + 7.$$

- An important thing to remember about a function is that its name refers to the whole association, not just to specific values. It is a “higher-order” object, not just a number. If you want to show two functions are equal, you have to show that they have the same domain and codomain, and that they return the same value for every value of the domain.

Venn diagrams of functions

