4-58.

a) P-value = P($\chi_0^2 > 22.35$):

for degrees of freedom of 14 we obtain 0.05 < P-value < 0.1

b) P-value = P($\chi_0^2 > 23.50$):

for degrees of freedom of 14 we obtain 0.05 < P-value < 0.1

c) P-value = P($\chi_0^2 > 25.00$):

for degrees of freedom of 14 we obtain 0.025 < P-value < 0.05

d) P-value = P($\chi_0^2 > 28.55$):

for degrees of freedom of 14 we obtain 0.01 < P-value < 0.025

4-59

a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.
The parameter of interest is the true standard deviation of the diameter, σ. However, the answer can be found by performing a hypothesis test on σ^2.
H_0: $\sigma^2 = 0.0004$
H_1: $\sigma^2 > 0.0004$
Test statistic is $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$
n = 15, s = 0.016

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{14(0.016)^2}{0.0004} = 8.96$$

P-value = P(χ^2 > 8.96) for 14 degrees of freedom: 0.5 < P-value < 0.9. Since the p-value > 0.05, we do not reject the null hypothesis.
b) 95% lower confidence interval on σ^2:

For α = 0.05 and n = 15, $\chi^2_{\alpha,n-1} = \chi^2_{0.05,14} = 23.68$

$$\frac{14(0.016)^2}{23.68} < \sigma^2$$

$0.00015 < \sigma^2$
With 95% confidence, we believe the true variance of the hole diameter is greater than 0.00015 mm^2. With 95% confidence, we believe the true standard deviation of the hole diameter is greater than 0.012 mm
c) Based on the lower confidence bound, we cannot reject the null hypothesis.

4-60

a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.
The parameter of interest is the true variance of the sugar content, σ^2.
H_0: $\sigma^2 = 18$

H_1: $\sigma^2 \neq 18$
$\alpha = 0.05$
$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi_{0.975,9} = 2.70$ or $\chi_0^2 > \chi^2_{\alpha/2,n-1}$ where $\chi^2_{0.025,9} = 19.02$

$n = 10$, $s^2 = 16$

$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(16)}{18} = 8$

Since $2.70 < 8 < 19.02$ do not reject H_0 and conclude the evidence indicates the true variance of the sugar content is not significantly different from 18 mg^2 at $\alpha = 0.05$.

b) P-value = $2P(\chi^2 \ > 8) \cong 1$ for 9 degrees of freedom

c) 95% confidence interval for σ:

First find a confidence interval for σ^2:

For $\alpha = 0.05$ and $n = 10$, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,9} = 19.02$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,9} = 2.70$

$\frac{9(16)}{19.02} \leq \sigma^2 \leq \frac{9(16)}{2.70}$

$7.57 \leq \sigma^2 \leq 53.33$

Take the square root of the endpoints of this interval to find the approximate confidence interval for σ: $2.75 \leq \sigma \leq 7.30$

With 95% confidence, we believe the true standard deviation of the sugar content is between 2.75 mg and 7.30 mg.

d) Since the hypothesized value lies within this confidence interval, the null hypothesis cannot be rejected.

4-62

a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

The parameter of interest is the true standard deviation of Izod strength, σ. However, the answer can be found by performing a hypothesis test on σ^2.

H_0: $\sigma^2 = 0.1$
H_1: $\sigma^2 \neq 0.1$
$\alpha = 0.01$
$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi^2_{0.995,19} = 6.84$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\chi^2_{0.005,19} = 38.58$

$n = 20$, $s = 0.328$

$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19(0.328)^2}{0.1} = 20.441$

Since $6.84 < 20.441 < 38.58$ we would not reject H_0 and conclude the true variance of Izod strength is not significantly different from 0.10 ft-lb/in at $\alpha = 0.01$.

b) P-value = $2P(\chi^2 > 20.441)$ for 19 degrees of freedom: $0.20 < 2P(\chi^2 > 20.441) < 1$

c) 99% confidence interval for σ^2 :

For $\alpha = 0.01$ and $n = 20$, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.005,19} = 38.58$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.995,19} = 6.84$

$$\frac{19(0.328)^2}{38.58} \le \sigma^2 \le \frac{19(0.328)^2}{6.84}$$

$0.053 \le \sigma^2 \le 0.299$

With 99% confidence, we believe the true variance of Izod strength is between 0.053 $(\text{ft-lb/in})^2$ and 0.299 (ft-lb/in).

d) Since the hypothesized value falls within this confident interval, we cannot reject the null hypothesis.

5-2

The parameter of interest is the difference in breaking strengths, $\mu_1 - \mu_2$ and $\Delta_0 = 10$

H_0: $\mu_1 - \mu_2 = 10$ or $\mu_1 = \mu_2$

H_1: $\mu_1 - \mu_2 > 10$ or $\mu_1 > \mu_2$

The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$\bar{x}_1 = 162.7$ $\bar{x}_2 = 155.4$ $\delta = 10$

$\sigma_1 = 1.0$ $\sigma_2 = 1.0$

$n_1 = 10$ $n_2 = 12$

$$z_0 = \frac{(162.7 - 155.4) - 10}{\sqrt{\frac{(1.0)^2}{10} + \frac{(1.0)^2}{12}}} = -6.31$$

P-value = $(1 - \Phi(-6.31)) = 1 - 0 \cong 1$. Since p-value is bigger than 0.05, we do not reject the null hypothesis.

5-3

a) The parameter of interest is the difference in mean burning rate, $\mu_1 - \mu_2$

H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

H_1: $\mu_1 - \mu_2 \ne 0$ or $\mu_1 \ne \mu_2$

$\alpha = 0.05$

The test statistic is