f) Energy Relationship

$$w(t) = \int_{-\infty}^{t} v(\tau)\, i(\tau)\, d\tau$$

$$= \int_{-\infty}^{v(t)} v(\tau)\, C \frac{dv}{d\tau}\, d\tau \rightarrow = C\int_{-\infty}^{v(t)} v(\tau)\, dv \rightarrow = C\left(\frac{v^2}{2}\right)^{T}$$

$$w(t) = \frac{1}{2} C v^2(t)$$

Passive element.

Terminal Conditions

$$i(t) = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v(t_0)$$

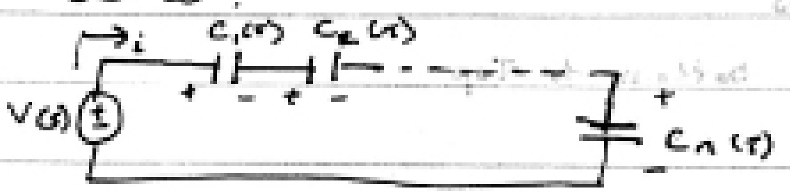
$$w_c(t) = \frac{1}{2}Cv^2(t)$$

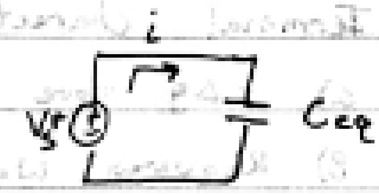
$\Delta q = C\Delta V$

$\boxed{q = CV}$

Equivalent Capacitance

series:





KVL: $v_s(t) = v_1 + v_2 + \dots v_n$

$$= \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v_1(t_0) + \int_{t_0}^{t} i(\tau)\, d\tau + v_2(t_0)$$

$$V_s(t) = \int_{t_0}^{t} i(t)\, dt + \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_n}\right] + (v_1 + v_2 \dots + v_n)$$

$$v_s(t) = \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)\, d\tau + v(t)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

$$\boxed{C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}}$$

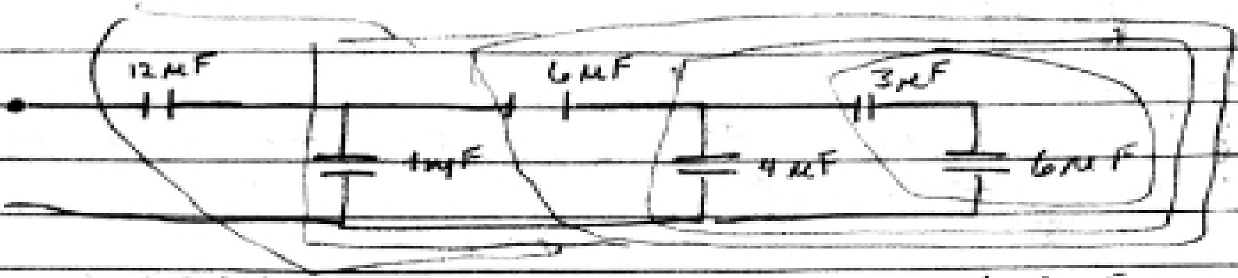
$$C = \frac{\epsilon A}{d}$$

Capacitance in Parallel

$$\boxed{C_{eq} = C_1 + C_2 + C_3 \dots + C_n}$$

$1F = \frac{1 \text{ coulomb}}{1 \text{ Volt}}$

Ex



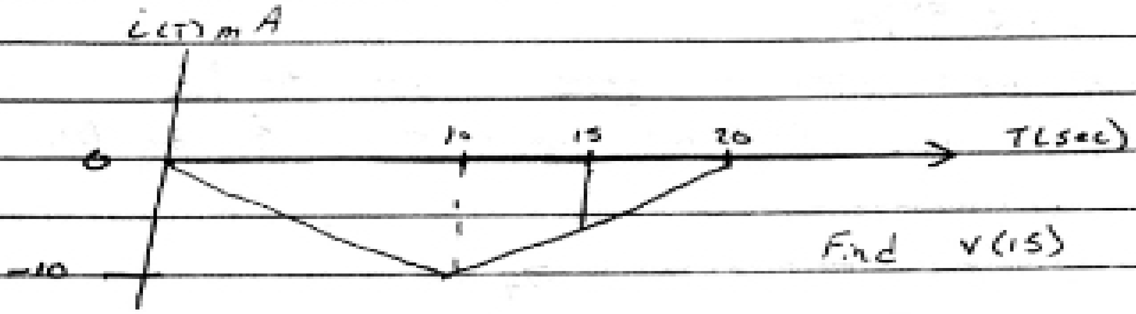
$\frac{3\mu F(6)}{9} = 2\mu F$

2 ∥ 4 $\quad 2+4 \quad 6\mu F$ $\quad$ 6 series 6 $\quad \frac{6(6)}{12} = 3\mu F$

$3\mu F \parallel 1\mu F = 4\mu F$

$4\mu F$ in series $12\mu F$ $= \frac{4(12)}{16} = 3\mu F$

A $10\mu F$ capacitor has $v(0) = 10V$ the current through the capacitor is given by:



Find $v(15)$

$$V(T) = \frac{1}{C}\int_{T_0}^{T} i(T)\,dT + V(T_0)$$

$-T \quad 0 \le T \le 10$ msec

$T - 20\times10^{-3} \quad 10\text{ msec} \le T \le 20\text{ msec}$

$$V(T) = \frac{1}{10\times10^{-6}}\left[\int_0^{10\times10^{-3}} -T\,dT + \int_{10\times10^{-3}}^{15\times10^{-3}} (T - 20\times10^{-3})\,dT\right] + V(0)$$

$$= \frac{1}{10\times10^{-6}}\left(-\frac{T^2}{2}\Big|_0^{10\times10^{-3}} + \left(\frac{T^2}{2} - 20\times10^{-3}T\right)\Big|_{10\times10^{-3}}^{15\times10^{-3}}\right) + 10V$$

$$= \frac{1}{10\times\cancel{10^{-6}}}\left(-50\times\cancel{10^{-6}} + \frac{125\times\cancel{10^{-6}}}{2} - 20\times\cancel{10^{-3}}(5\times\cancel{10^{-3}})\right) + 10V$$

$$= \frac{1}{10}(-50 + 62.5 - 100) + 10$$

$$= -8.75 + 10 = \boxed{+1.25V}$$

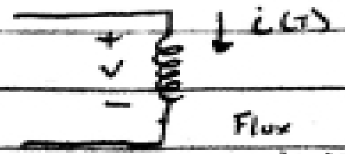
or

$$\frac{1}{10\times\cancel{10^{-6}}}\left(\tfrac{1}{2}(10\times\cancel{10^{-3}})(-10\times\cancel{10^{-3}}) + \left(\frac{-15\times\cancel{10^{-3}}}{2}\right)(5\times\cancel{10^{-3}})\right) + 10$$

$$\frac{1}{10}(-50 - 37.5) + 10$$

$$-8.75 + 10 = \boxed{1.75V}$$

B. Inductor [coil]



Flux, ϕ — ϕ increase as i increase

Flux linkage $\rightarrow \lambda = N\phi$

$\lambda \sim i$

$\lambda = Li$ — L: Inductance [Henry] H

$\lambda = Li$ — $\Delta\lambda = L\Delta i$

$\frac{\Delta\lambda}{\Delta T} = L\frac{\Delta i}{\Delta T}$ — $\lim_{\Delta T \to 0} \frac{\Delta\lambda}{\Delta T} = L \lim_{\Delta T \to 0} \frac{\Delta i}{\Delta T}$

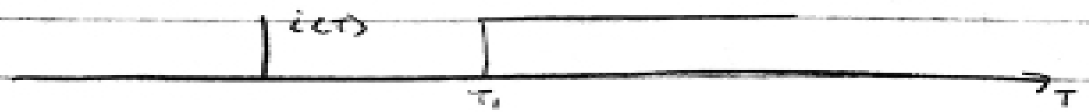
$\frac{d\lambda}{dT} = L\frac{di}{dT}$

Faraday's law of Electromagnetic induction: $\frac{d\lambda}{dT} = V$

$\boxed{V = L\frac{di}{dT}}$ — $i = C\,dv/dT$

An inductor acts like a short circuit to DC

A Capacitor acts like an open circuit to DC



The Current Through an inductor cannot change instantaneously

$\frac{di}{dT} = \frac{1}{L}V_1$

$\int_{i(T_0)}^{i} di = 1/L \int_{T_0} v(T)\,dT \rightarrow i(T) - i(T_0) = 1/L \int_{T_0}^{T} v(T)\,dv$

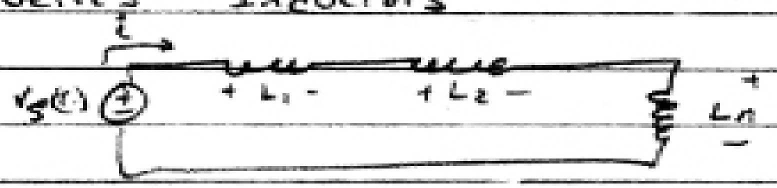
$i(T) = 1/L \int_{T_0}^{T} v(T)\,dv + i(T_0)$

Energy Relationship.

$w(T) = \int_{-\infty}^{i(T)} v(T)\,(i(T))\,dT = \int_{-\infty}^{i(T)} L\frac{di}{\cancel{dT}}\,i(T)\,\cancel{dT}$

$w(T) = L\frac{i^2}{2} = 1/2\,L\,i^2(T) \geq 0$ — Passive element.

Series Inductors



KVL: $V_s(T) = V_1 + V_2 \cdots V_n$

$= L_1\frac{di}{dT} + L_2\frac{di}{dT} + \cdots + L_n\frac{di}{dT}$

$= \frac{di}{dT}[L_1 + L_2 + L_n]$

$V_s(T)$

Series $L_{eq} = L_1 + L_2 + L_n$ — Act just like Resistors

Parallel $L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_n}}$