

## Outline of COT 3100 material for first exam

### I. Counting

- A. Sum Rule
- B. Product Rule
- C. Subtraction Idea
- D. Permutations
- E. Combinations
- F. Combinations with repetition

### II. Logic

- A. Symbols( $\neg$ ,  $\wedge$ , and  $\vee$ )
- B. Truth Tables
- C. Logic Laws
- D. Methods of showing equality of logical expressions
- E. Implication Rules
- F. Contrapositive of a stmt.

### III. Sets

- A. Symbols( $\cup$ ,  $\cap$ ,  $\setminus$ ,  $\subseteq$ ,  $\supseteq$ , and  $\equiv$ )
- B. Counting with sets
- C. Set Laws
- D. Membership Table
- E. Proof Techniques for if-then statements
  - i. direct proof
  - ii. proof of contrapositive
  - iii. proof by contradiction
- F. How to Disprove an if-then statement
- G. Inclusion-Exclusion Principle

Reading in textbook: 1.1 – 1.4, 2.1 – 2.3, 3.1 – 3.3

1) Find the number of permutations of letters of the English alphabet (i.e. strings of length 26 using each letter exactly once) that contain none of the strings *math*, *bio* or *housing*.

**Total number of permutations = 26!**

**Permutations with math = 23!**

**Permutations with bio = 24!**

**Permutations with housing = 20!**

**Permutations with both math and bio = 21!**

**Permutations with both math and housing = 17!**

**Permutations with both bio and housing = 0**

**Permutations with math, bio and housing = 0**

$$\begin{aligned}\text{Total} &= 26! - (23! + 24! + 20! - (21! + 17!) + 0) \\ &= 26! - 24! - 23! - 20! + 21! + 17!\end{aligned}$$

2) In how many ways can 3 blue, 4 white and 2 red balls be distributed into 4 distinct boxes?

**Distributing each the blues balls is independent of distributing the white ones and the red ones. Using the technique in section 1.4 we know we can distribute 3 blues balls amongst 4 distinguishable boxes in  ${}_{4-1+3}C_3 = 20$  ways. Similarly, we can distribute the white balls in  ${}_{4-1+4}C_4 = 35$  ways and the red balls in  ${}_{4-1+2}C_2 = 10$  ways. Using the product rule, we can distribute the balls in  $20 \times 35 \times 10 = 7000$  ways.**

3). In a language survey of students it is found that 80 students know English, 60 know French, 50 know German, 30 know English and French, 20 know French and German, 15 know English and German and 10 students know all three languages.

How many students know

- at least one language?
- English only?
- French and one but not both out of English and German?

d) at least two languages?

**Use the inclusion-exclusion principle to handle each of these questions.**

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| \\ &\quad - |A \cap B| - |B \cap C| - |A \cap C| \\ &\quad + |A \cap B \cap C|. \end{aligned}$$

**Let A = students who speak English, B = students who speak French, and C = students that speak German**

$$|A \cup B \cup C| = 80 + 60 + 50 - 30 - 20 - 15 + 10 = 135 \text{ total students that know at least one language.}$$

**Now, let's count the number of students that know English and at least one other language, using the inclusion exclusion principle.**

$$|A \cap B| + |A \cap C| - |A \cap B \cap C| = 30 + 15 - 10 = 35.$$

**Thus, the number of students that just speak English is simply  $80 - 35 = 45$ .**

**Total that speak French and another language is equal to**

$$|A \cap B| + |B \cap C| - |A \cap B \cap C| = 30 + 20 - 10 = 40.$$

**Of these students, 10 of them speak 2 different languages. You want to subtract these out also, so there is a total 30 that speak French and exactly one other language.**

**The number of people that speak at least 2 languages is simply**