

**2.1 (due Sep 21, 2006)** There are 20 people at a party. Each person writes her/his name on a card (suppose that all the people in the room have different names). The cards are then mixed and each person is given back a card.

- a) What is the expected number of people that get their card back?
- b) What is the probability that exactly one person gets his/her card back? (Evaluate your answer to 3 digits of accuracy, your answer should be of the form  $0.xyz$ , where  $x, y, z$  are decimal digits.)

$X_i$  = indicator random variable of the event that  $i$ -th person gets his hat back.

$$E[X_i] = 1/20$$

$$X = X_1 + \dots + X_{20}$$

$$E[X] = E[X_1] + \dots + E[X_{20}] = 1$$

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$$n \lfloor (n-1)! / e \rfloor$$

$$\frac{n \lfloor (n-1)! / e \rfloor}{n!} \approx 0.3678 \approx 0.368$$