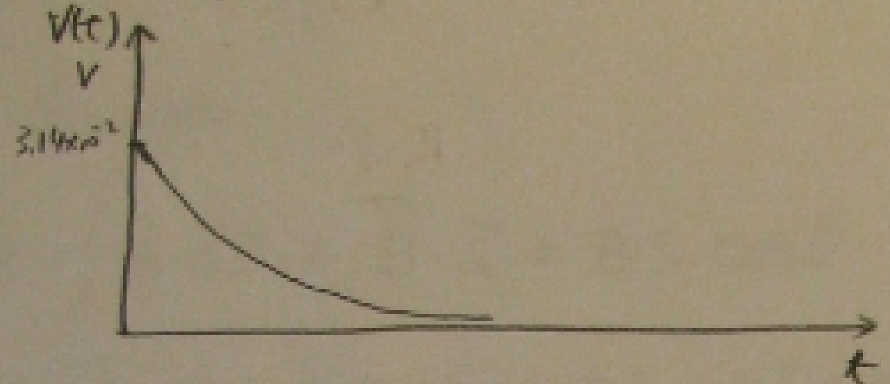
HW 8

5.1.2 $\phi = \int_s \vec{B}\cdot ds = 10e^{-t/10}\cdot \pi(0.1)^2$

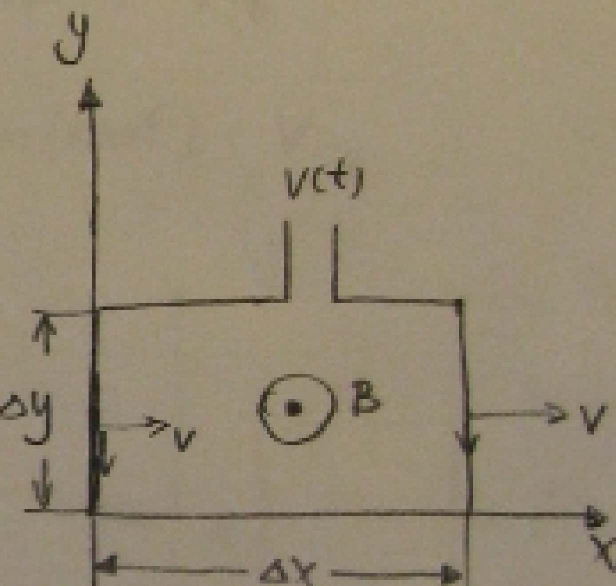
$= 0.1\pi e^{-t/10}$

$V_{emf} = -\frac{d\phi}{dt} = -0.1\pi\cdot(-\frac{1}{10})e^{-t/10} = 0.01\pi e^{-t/10}$ V

$= 3.14\times10^{-2}e^{-t/10}$ V



5.1.3.



$V = \int(\vec{V}\times\vec{B})\cdot d\vec{l}$

Only left and right sides of the rectangle have none zero $V = \int \vec{V}\times\vec{B}\cdot d\vec{l}$.

Since $\vec{B} = y\hat{u}_z$ is only dependent on y.

The voltage at these two side are the same. $V(t) = 0$

Or in othere word, as the close loop move in $\hat{u}_x$ direction there is no magnetic flux change.

Intrested student can calculate $V(t)$ when $\vec{V}_0 = 5\hat{u}_y$

5.1.4.

$V(t) = 0$ The magnetic flux enclosed by the loop dose not change as it moves

5.1.5

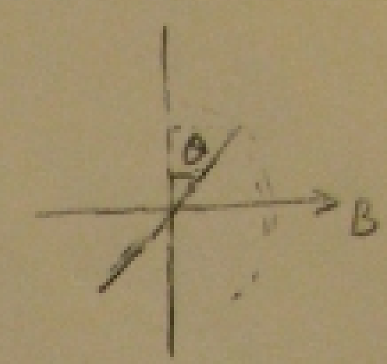
$\vec{V} = v\hat{u}_x = 3\hat{u}_x$ m/s

$\vec{B} = 5\hat{u}_z$ T

$$V = \int_0^L \vec{V}\times\vec{B}\cdot d\vec{l} = \int_0^L 3\hat{u}_x \times 5\hat{u}_z \cdot dl$$

$$= -15 \cdot L = -6V$$

5.1.10



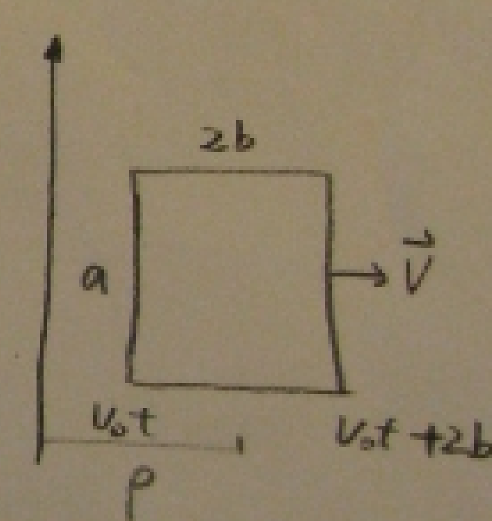
$\theta = \omega t$

$$\phi = \int \hat{B}\cdot d\vec{s} = \vec{B}\cdot\vec{S} = B\cdot S\cdot\cos\theta$$

$$= 2\times 0.5\times 0.5\cdot\cos(1200t) = 0.5\times\cos(1200t) \ W_b$$

$$V = -\frac{d\phi}{dt} = +\ 0.5\times 1200 \sin(1200t) = 600\sin(1200t) \ V$$

5.1.11



$$\vec{B} = \hat{u}_\phi \frac{\mu_0 I}{2\pi\rho}$$

$$\phi = \int \vec{B}\cdot d\vec{s} = \int_{V_0t}^{V_0t+2b} \frac{\mu_0 I}{2\pi\rho}\cdot a\, d\rho$$

$$= \frac{\mu_0 I a}{2\pi}\int_{V_0t}^{V_0t+2b} \frac{1}{\rho}\, d\rho = \frac{\mu_0 I a}{2\pi}\left[\ln(V_0t+2b) - \ln(V_0t)\right]$$

$$V = -\frac{d\phi}{dt} = \frac{-\mu_0 I a}{2\pi}\left(\frac{V_0}{V_0t+2b} - \frac{V_0}{V_0t}\right)$$

$$= \frac{\mu_0 I\, 2ab}{2\pi}\,\frac{1}{(V_0t+2b)t} = \frac{\mu_0 I ab}{\pi(V_0t+2b)t}$$

5.1.12

$$\Phi = \int \vec{B}\cdot d\vec{s}$$

$$= 0.5 \cdot \cos\phi\, \hat{u}_\phi \cdot r \cdot L\, \hat{u}_\phi$$

$$= 0.5 \cdot 0.2 \times 1 \cos\phi = 0.1\cos\phi = 0.1\cos(\omega t) \quad (W_b)$$

$$V = -\frac{d\phi}{dt} = 0.1\omega \sin\omega t = 4\pi \sin(40\pi t) \quad (V)$$

$$I = \frac{V}{R} = \frac{4\pi}{100}\sin(40\pi t) \simeq 0.126 \sin(40\pi t) \quad (A)$$

5.3.1 $\vec{J}_c = \sigma \vec{E}$ $\quad \vec{J}_d = \frac{\partial \vec{D}}{\partial t} = j\omega\varepsilon \vec{E}$

$$\frac{|\vec{J}_c|}{|\vec{J}_d|} = \frac{\sigma}{\omega\varepsilon}$$

f \ mate.	Copper	Sea water	earth
60Hz	1.74×10^{16}	1.48×10^{7}	3.00×10^{4}
1MHz	1.04×10^{12}	8.88×10^{2}	1.80
1GHz	1.04×10^{9}	0.888	0.0018

5.3.2

$$|J_d| = \frac{\omega\varepsilon}{\sigma}|J_c| = \frac{2\pi\times10^9\times65\times8.854\times10^{-12}}{10^{-3}} \times 0.2 \cdot \sin(2\pi\times10^9 t)$$

$$= 72.32 \sin(2\pi\times10^9 t) \ A/m^2$$