$\boxed{\text{Section 7.2}}$ Partial Derivatives

A partial derivative of a function of more than one variable is a rate of change that measures how fast the function is changing as a variable increases, while the other variable(s) remain constant (or fixed)

*Partial derivative Notation and operator

For $z = f(x,y)$

$\frac{\partial f}{\partial x} = \frac{\partial z}{\partial x} = f_x$ Partial derivative of the function with respect to x

$\frac{\partial f}{\partial y} = \frac{\partial z}{\partial y} = f_y$ Partial derivative of the function with respect to x

Operator: $\frac{\partial}{\partial x}\left[f(x,y)\right] = \frac{\partial f}{\partial x}$

$$\frac{\partial}{\partial y}\left[f(x,y)\right] = \frac{\partial f}{\partial y}$$

Example: For $f(x,y) = 4x^3y^2 + 2xy + 4x$
Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\left.\frac{\partial f}{\partial x}\right|_{(1,2)}$, and $\left.\frac{\partial f}{\partial y}\right|_{(1,2)}$

$$\frac{\partial f}{\partial x} = 12x^2y^2 + 2y + 4 = f_x$$

$$\frac{\partial f}{\partial y} = 8x^3y + 2x = f_y$$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,2)} = f(x(1,2)) = 12(1)^2(2)^2 + 2(2) + 4 = 56$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,2)} = f(y(1,2)) = 8(1)^3(2) + 2(1) = 18$$

Higher Order Partial Derivatives

The second partial with respect to the same variable

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \boxed{\frac{\partial^2 f}{\partial x^2} = f_{xx}}$$

or

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \boxed{\frac{\partial^2 f}{\partial y^2} = f_{yy}}$$

The second partial with respect to the other variable

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \boxed{\frac{\partial^2 f}{\partial y \partial x} = f_{xy}}$$

$$\frac{\partial x}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \boxed{\frac{\partial^2 f}{\partial x \partial y} = f_{yx}}$$

Note: $f_{xy} = f_{yx}$

Example: For $f(x,y) = 4x^3y^2 + 2xy + 4x$

Find: $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, and $\frac{\partial^2 f}{\partial y \partial x}$

Recall: $\frac{\partial f}{\partial x} = 12x^2y^2 + 2y + 4$

$\frac{\partial f}{\partial y} = 8x^3y + 2x$

$\frac{\partial^2 f}{\partial x^2} = 24xy^2 = f_{xx}$

$\frac{\partial^2 f}{\partial y^2} = 8x^3$

$\frac{\partial^2 f}{\partial x \partial y} = 24x^2y + 2$

$\frac{\partial^2 f}{\partial y \partial x} = 24x^2y + 2$