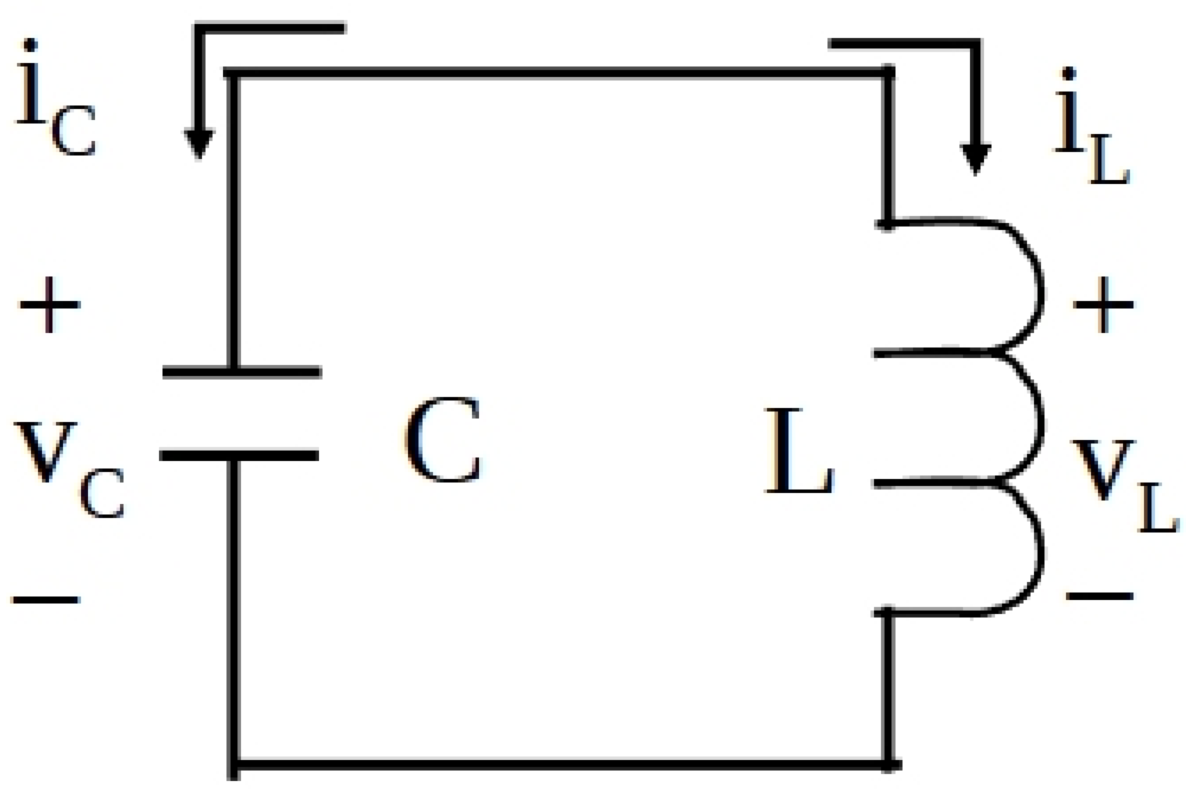
Introduction to Second Order Circuits

LC Circuits

Combining an inductor and capacitor.



KVL: $v_C(t) = v_L(t)$

KCL: $i_c(t) = -i_L(t)$

$$-i_L(t) = i_c(t) = C\frac{dv_c(t)}{dt} \qquad (22.1)$$

Recall:

$$\frac{di_L(t)}{dt} = \frac{1}{L} v_L(t)$$

Take derivative of Eqn. (22.1)

$$\frac{-di_L(t)}{dt} = \frac{-1}{L} v_c(t) = C\frac{d^2v_c(t)}{dt^2}$$

$$\frac{d^2 v_c(t)}{dt^2} = \frac{-1}{LC} v_c(t) \qquad (22.2)$$

Similarly,

$$L \frac{di_L(t)}{dt} = v_L(t) = v_C(t)$$

Differentiate w.r.t time,

$$\frac{d^2 i_L(t)}{dt^2} = \frac{1}{L} \frac{dv_c(t)}{dt} = \frac{1}{LC} i_c(t)$$

$$\frac{d^2 i_L(t)}{dt^2} = \frac{-1}{LC} i_L(t) \qquad (22.3)$$

Eqns. 22.2 and 22.3 are often written as:

$$\frac{d^2v_c(t)}{dt^2} + \frac{1}{LC}v_c(t) = 0 \qquad (22.2a)$$

$$\frac{d^2i_L(t)}{dt^2} + \frac{1}{LC}i_L(t) = 0 \qquad (22.3a)$$

General differential equation for LC circuits

$$\frac{d^2x(t)}{dt^2} + \frac{1}{LC}x(t) = 0 \qquad (22.4)$$

Eqn. 22.4 describes the behavior of the capacitor voltage or inductor current in an LC circuit.

Possible solutions for Eqn.(22.4)

$$x(t) = A\cos(\omega t)$$

$$x(t) = B\sin(\omega t)$$

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

All valid for specific values of A, B, and ω.