

- 1) Estimate the area under the graph of  $f(x) = x^2 - 2x$  from  $x = 0$  to  $x = 8$  using 4 approximating rectangles and midpoints (i.e. using  $M_4$ ).  
 A) 100 B) 101 C) 102 D) 103 \*E) 104 F) 105 G) 106 H) 107 I) 108 J) 109

Solution:  $\Delta x = 2$ , and midpoints are 1, 3, 5, 7. Then

$$M_4 = 2(f(1) + f(3) + f(5) + f(7)) = 2(-1 + 3 + 15 + 35) = 104. \text{ (E)}$$

- 2) Which of the following is a **Riemann sum** for the area under  $f(x) = x^2$ ,  $1 \leq x \leq 2$ , using the **right endpoints**.

A)  $\frac{1}{n} \sum_{i=0}^{n-1} (2 + \frac{i}{n})^2$  B)  $\frac{1}{n} \sum_{i=1}^n (2 + \frac{i}{n})^2$  C)  $\frac{2}{n} \sum_{i=0}^{n-1} (1 + \frac{i}{n})^2$  \*D)  $\frac{1}{n} \sum_{i=1}^n (1 + \frac{i}{n})^2$   
 E)  $\frac{1}{n} \sum_{i=1}^n (1 + (\frac{i}{n})^2)$  F)  $\frac{1}{n} \sum_{i=1}^n (\frac{i+1}{n})^2$  G)  $\frac{2}{n} \sum_{i=0}^{n-1} (1 + \frac{i+1}{n})^2$  H)  $\frac{1}{n} \sum_{i=0}^{n-1} (\frac{i+1}{n})^2$   
 I)  $\frac{1}{n} \sum_{i=1}^n (2 + \frac{i+1}{n})^2$

Solution:  $\Delta x = \frac{b-a}{n} = \frac{1}{n}$ . First point in Riemann Sum should be  $(1 + \frac{1}{n})^2 (\frac{1}{n})$ .  
 The only sum, above, fulfilling these properties is (D).

- 3) Find the average value of the function  $f(t) = t^2 - t$ , on  $[-2, 1]$ .  
 A) 0.25 B) 0.5 C) 0.75 D) 1 E) 1.25 \*F) 1.5 G) 1.75 H) 2 I) 2.25 J) 2.5

Solution:  $av(f) = \frac{1}{1-(-2)} \int_{-2}^1 (t^2 - t) dt = \frac{1}{3} (\frac{t^3}{3} - \frac{t^2}{2}) \Big|_{-2}^1 = \frac{3}{2}$ . (F)

- 4)  $\int_0^6 \frac{1}{4} x^2 dx = \lim_{n \rightarrow \infty} R_n$ , where  $R_n = \sum_{k=1}^n f(c_k) \Delta x$ , is the Riemann sum with  $c_k$  being the right endpoint. Using the formula,  $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$ , we get  $R_n =$

A)  $3 \frac{n(n+1)(2n+1)}{n^3}$  B)  $6 \frac{n(n+1)(2n+1)}{n^3}$  C) \*9  $\frac{n(n+1)(2n+1)}{n^3}$  D)  $12 \frac{n(n+1)(2n+1)}{n^3}$   
 E)  $16 \frac{n(n+1)(2n+1)}{n^3}$  F)  $18 \frac{n(n+1)(2n+1)}{n^3}$  G)  $24 \frac{n(n+1)(2n+1)}{n^3}$  H)  $28 \frac{n(n+1)(2n+1)}{n^3}$

Solution:  $\frac{b-a}{n} = \frac{6}{n}$ , and  $x_k = \frac{6k}{n}$ . So  $f(x_k) = \frac{1}{4} (\frac{6k}{n})^2$ . Then  $\sum_{k=1}^n f(x_k) \frac{b-a}{n} =$

$$\frac{1}{4} \cdot \frac{36}{n^2} \cdot \frac{6}{n} \sum_{k=1}^n k^2 = \frac{54}{n^3} \frac{n(n+1)(2n+1)}{6} = 9 \frac{n(n+1)(2n+1)}{n^3}. \text{ (C)}$$

5) The indefinite integral  $\int \frac{e^z}{1+e^{2z}} dz = F(z) + C$ , where  $F(z) =$ :

- A)  $e^{3z}$  B)  $\frac{1}{1+e^{2z}}$  C)  $\sin^{-1}(2z)$  D)  $\ln(1 + e^{2z})$  E)  $\tan^{-1}(z)$  F)  $\sqrt{1 + e^z}$   
 G)  $(1 + e^{2z})^{-2}$  \*H)  $\tan^{-1}(e^z)$  I)  $1 + e^z$  J)  $(1 + e^z)^{-2}$

Solution: ( $u = e^z$ ,  $du = e^z dz$ )  $= \int \frac{1}{1+u^2} du = \tan^{-1}(u) + C = \tan^{-1}(e^z)$ . (H)

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6) The general indefinite integral  $\int \frac{\sin(t)}{1+\tan^2(t)} dt$ , has a solution  $F(t) + C$ , where  $F(t) =$ :

- A)  $\sec(t)$  B)  $\frac{\sec^2(t)}{2}$  C)  $-\frac{\sec^3(t)}{3}$  D)  $\sin(t)$  E)  $-\frac{\sin^2(t)}{2}$   
 F)  $\frac{\sin^3(t)}{3}$  G)  $\tan(t)$  H)  $-\frac{\tan^3(t)}{2}$  I)  $\frac{\cos^3(t)}{2}$  \*J)  $-\frac{\cos^3(t)}{3}$

Solution: (by trig identity)  $= \int \frac{\sin(t)}{\sec^2(t)} dt = \int \cos^2(t) \sin(t) dt = -\frac{\cos^3(t)}{3} + C$ . (J)  
 (This can follow from substitution,  $u = \cos(t)$ ,  $du = -\sin(t) dt$ .)

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7) Use Part 1 of the Fundamental Theorem of Calculus to find  $\frac{d}{dx} \int_0^{x^2} \tan(t) dt$ .

- A)  $2x \sec^2(x^2)$  B)  $\frac{\sec^2(x^2)}{x^2}$  C)  $\frac{\tan(x^2)}{2x} - \frac{\tan(x^{\frac{1}{2}})}{x^{\frac{1}{2}}}$  D)  $\frac{\sec^2(x^2)}{x^3}$  E)  $\frac{\tan(x^2)}{x^2}$   
 F)  $x^2 \tan(x^2)$  \*G)  $2x \tan(x^2)$  H)  $2x \tan(x^2)$  I)  $x \tan(x^2)$

Solution:  $2x \tan(x^2)$ . (G)

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8) Using the substitution  $u = 4x^3 + 3x^2$ , write the integral  $\int_0^1 \frac{2x^2+x}{(4x^3+3x^2)^2} dx$ ,  
 as an integral in the variable  $u$ .

- A)  $\int_0^1 \frac{du}{u^2}$  B)  $3 \int_0^1 \frac{du}{u^2}$  C)  $6 \int_0^3 \frac{du}{u^2}$  D)  $\frac{1}{3} \int_0^3 \frac{du}{u^2}$  E)  $\frac{1}{6} \int_0^3 \frac{du}{u^2}$  F)  $\int_0^6 \frac{du}{u^2}$  G)  $\frac{1}{3} \int_0^6 \frac{du}{u^2}$   
 \*H)  $\frac{1}{6} \int_0^7 \frac{du}{u^2}$  I)  $6 \int_0^7 \frac{du}{u^2}$  J)  $\frac{1}{3} \int_0^7 \frac{du}{u^2}$

Solution:  $du = (12x^2 + 6x) dx = 6(2x^2 + x) dx \Rightarrow (2x^2 + x) dx = \frac{1}{6} du$ . Therefore  
 $\int_0^1 \frac{2x^2+x}{(4x^3+3x^2)^2} dx = \frac{1}{6} \int_0^7 \frac{du}{u^2}$ . (H)

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9) Evaluate the integral  $\int_1^2 \frac{x^3+x^2-1}{x^3} dx$ .

- A)  $\frac{1}{2}$  B) 1 C)  $\frac{3}{2}$  \*D) 2 E)  $\frac{5}{2}$  F) 3 G)  $\frac{7}{2}$  H) 4 I)  $\frac{9}{2}$  J) 5

Solution:  $\int_1^2 \frac{x^3+x^2-1}{x^3} dx = \int_1^2 (x + 1 - x^{-2}) dx = \frac{x^2}{2} + x + \frac{1}{x} \Big|_1^2 = \frac{9}{2} - \frac{5}{2} = 2$ . (D)

10) Evaluate the integral  $\int_0^3 |x - 2| dx$ .

A)  $\frac{1}{2}$  B)  $\frac{3}{2}$  \*C)  $\frac{5}{2}$  D)  $\frac{7}{2}$  E)  $\frac{9}{2}$  F)  $\frac{3}{4}$  G)  $\frac{7}{4}$  H)  $\frac{11}{4}$  I)  $\frac{15}{4}$  J)  $\frac{19}{4}$

Solution:  $= \int_0^2 (2 - x) dx + \int_2^3 (x - 2) dx = 2 + \frac{1}{2} = \frac{5}{2}$ . (C)

11) The indefinite integral  $\int \frac{t}{\sqrt{t-2}} dt = F(t) + C$ , where  $F(t) =$

\*A)  $\frac{2}{3}(t-2)^{\frac{3}{2}} + 4(t-2)^{\frac{1}{2}}$  B)  $2(t-2)^{\frac{1}{2}} - (t-2)^{-\frac{1}{2}}$  C)  $(t-2)^{\frac{1}{2}} + (t-2)^{-\frac{1}{2}}$   
 D)  $\frac{2}{3}(t-2)^{\frac{3}{2}} - 2(t-2)^{\frac{1}{2}}$  E)  $2\sqrt{t-2}$  F)  $t\sqrt{t-2}$  G)  $\frac{1}{2}\sqrt{t-2}$  H)  $t + \sqrt{t-2}$   
 I)  $\frac{2}{3}t^{\frac{3}{2}} + 4t^{\frac{1}{2}}$  J)  $\frac{3}{2}\sqrt{t} + \frac{4}{\sqrt{t-2}}$

Solution: ( $u = t - 2$ ,  $du = dt$ ,  $t = u + 2$ )  $= \int \frac{u+2}{\sqrt{u}} du = \int u^{\frac{1}{2}} + 2u^{-\frac{1}{2}} du =$   
 $\frac{2}{3}u^{\frac{3}{2}} + 4u^{\frac{1}{2}} + C = \frac{2}{3}(t-2)^{\frac{3}{2}} + 4(t-2)^{\frac{1}{2}} + C$ . (A)

12) Find a solution for the indefinite integral  $\int \frac{ax+b}{(ax^2+2bx+c)^3} dx$ , where  $a$ ,  $b$ , and  $c$  are non-zero constants.

A)  $\frac{1}{(ax^2+2bx+c)^2}$  B)  $\frac{2}{(ax^2+2bx+c)^2}$  C)  $\frac{-1}{(ax^2+2bx+c)^2}$  D)  $\frac{-2}{(ax^2+2bx+c)^2}$   
 E)  $\frac{1}{4(ax^2+2bx+c)^2}$  \*F)  $\frac{-1}{4(ax^2+2bx+c)^2}$  G)  $\frac{1}{(ax^2+2bx+c)^4}$   
 H)  $\frac{-1}{(ax^2+2bx+c)^4}$  I)  $\frac{2}{(ax^2+2bx+c)^4}$  J)  $\frac{-2}{(ax^2+2bx+c)^4}$

Solution: ( $u = ax^2 + 2bx + c$ ,  $du = (2ax + 2b) dx = 2(ax + b) dx$ ). Therefore  
 $\int \frac{ax+b}{(ax^2+2bx+c)^3} dx = \int \frac{1}{2}u^{-3}du = -\frac{1}{4u^2} + C = -\frac{1}{4(ax^2+2bx+c)^2} + C$  (F)

13) Find the area of the region enclosed by the line  $x = y + 2$ , and the curve  $x = y^2$ .

A)  $\frac{1}{2}$  B)  $\frac{3}{2}$  C)  $\frac{5}{2}$  D)  $\frac{7}{2}$  \*E)  $\frac{9}{2}$  F)  $\frac{11}{2}$  G)  $\frac{13}{2}$  H)  $\frac{15}{2}$  I)  $\frac{17}{2}$  J)  $\frac{19}{2}$

Solution: Solving  $y^2 = y + 2$ , we get  $y = -1$  ( $x = 1$ ), and  $y = 2$  ( $x = 4$ ). So the points of intersection are,  $(1, -1)$ ,  $(4, 2)$ . The parabola  $x = y^2$ , has its vertex at the origin and opens to the right, and in that region,  $x = y + 2$ , is the boundary, to the right. Therefore area is,  $\int_{-1}^2 (y + 2) - y^2 dy = \frac{9}{2}$ . (E)

14) Find the area of the region enclosed by the curves  $y = 7 - 2x^2$  and  $y = x^2 + 4$ .

A)  $\frac{1}{2}$  B) 1 C)  $\frac{3}{2}$  D) 2 E)  $\frac{5}{2}$  F) 3 G)  $\frac{7}{2}$  \*H) 4 I)  $\frac{9}{2}$  J) 5

Solution: Solving  $7 - 2x^2 = x^2 + 4$ , we get the intersection points being,  $(\pm 1, 5)$ . The parabola  $y = 7 - 2x^2$  is above the parabola  $y = x^2 + 4$ , so the area is given by  
 $\int_{-1}^1 (7 - 2x^2) - (x^2 + 4) dx = \int_{-1}^1 (3 - 3x^2) dx = 2\int_0^1 (3 - 3x^2) dx = 4$  (H)