MATH 132 **EXAM II (solution)** FALL 2012

1) Find the length of the curve $y = \frac{1}{3}(x^2-2)^{3/2}$, from $x = 3$ to $x = 5$.

$A)$ 28.66 $B)$ 30.66 $C)$ 32.66 $D)$ 34.66 $E)$ 36.66 $F)$ 38.66 $G)$ 40.66
$H)$ 42.66 $I)$ 44.66 $J)$ 46.66

solution: $\frac{dy}{dx} = \frac{1}{2}(x^2-2)^{\frac{1}{2}}(2x) = x(x^2-2)^{\frac{1}{2}}$, $(\frac{dy}{dy})^2 = x^2(x^2-2) = x^4-2x^2$,
$1+(\frac{dy}{dx})^2 = x^4-2x^2+1 = (x^2-1)^2$, $\sqrt{1+(\frac{dy}{dx})^2} = x^2-1$.
$l = \int_3^5 x^2-1\, dx = \frac{92}{3} \sim 30.66$ (B)

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2) The curve $y = \sqrt{x}$, $\frac{3}{4} \leq x \leq \frac{15}{4}$, is revolved about the $x-axis$, find the surface area of the solid gerenrated by this proceedure.

$A)$ $\frac{2\pi}{3}$ $B)$ $\frac{8\pi}{3}$ $C)$ $\frac{16\pi}{3}$ $D)$ $\frac{22\pi}{3}$ $E)$ $\frac{28\pi}{3}$ $F)$ $\frac{34\pi}{3}$ $G)$ $\frac{38\pi}{3}$ $H)$ $\frac{41\pi}{3}$ $I)$ $\frac{46\pi}{3}$ $J)$ $\frac{49\pi}{3}$

solution: $1+(\frac{dy}{dx})^2 = 1+\frac{1}{4x} = \frac{4x+1}{4x}$. $S = 2\pi\int_{3/4}^{15/4} \sqrt{x}\sqrt{\frac{4x+1}{4x}}\, dx =$
$\pi\int_{3/4}^{15/4}\sqrt{4x+1}\, dx = \pi\frac{1}{4}\frac{2}{3}(4x+1)^{3/2}\Big|_{3/4}^{15/4} = \frac{\pi}{6}(64-8) = \frac{28\pi}{3}$ (E)

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3) If a force of 90 N stretches a spring 1 m beyond its natural length, how much work, in J, does it take to stretch the spring 5 m beyond its natural length?

$A)$ 1125 $B)$ 1175 $C)$ 1225 $D)$ 1275 $E)$ 1325 $F)$ 1375
$G)$ 1425 $H)$ 1475 $I)$ 1525 J) 1575

solution: Given, $F(1) = k\cdot 1 = 90 \Rightarrow F(x) = 90x$. $W = \int_0^5 90x\, dx = 1125$. (A)

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4) A heavy uniform cable is used to lift a 300 lb load from ground level to the top of a building that is 120 ft high. If the cable weighs 40 lb/ft, then how much work, in thosands of ft-lb, is done?

A) 122 B) 186 C) 244 D) 282 E) 324 F) 362 G) 488 H) 536 I) 584 J) 622

solution: The original weight is $300 + (40)(120) = 5100\, lb$. After lifting y feet, we have $F(y) = 5100 - 40y$, since y ft of the cable is already lying on the roof. So $W = \int_0^{120} 5100 - 40y\, dy = 324,000\ ft-lb$. (E)

5) A wading pool has rectangular base with side lengths of 6 and 10 ft. The pool is 4 ft deep, but the water in it is only 3 ft high. If the water weighs 62.4 lb/ft^3, how much work, in ft-lb, is done in pumping all the water out, over the top of the pool?

$A)$ 20367 $B)$21765 $C)$ 22422 $D)$ 23762 $E)$ 24482 $F)$ 25386 $G)$ 26842
$H)$ 27654 $I)$ 28080 $J)$ 29952

solution: Area of cross-section at level, y, is 60 ft^2 and volume of slab is 60 Δy ft^3. Weight of each slab is (62.4)(60 Δy) = 3744 lb. The slab needs to be lifted $4 - y\ ft$. The water level is from 0 to 3. Therefore the work is given by,
$W = 3744 \int_0^3 (4 - y)\, dy \;=\; 28080\ ft - lb.$ (I)

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6) Given $x\,\frac{dy}{dx} = ln(x)$, and $y(e) = 2$, we have that $y(1) =$:

$A)\,0$ $B)\,\frac{1}{2}$ $C)\,1$ $D)\,\frac{3}{2}$ $E)\,2$ $F)\,\frac{5}{2}$ $G)\,3$ $H)\,4$ $I)\,\frac{7}{2}$ $J)\,5$

solution: $\frac{dy}{dx} = \frac{ln(x)}{x}$, then $y = \frac{1}{2}\,ln^2(x) + C$ ($u = ln(x),\ du = \frac{1}{x}\,dx$).
$2 = y(e) = \frac{1}{2}\,ln^2(e) + C = \frac{1}{2} + C \Rightarrow C = \frac{3}{2}$. Therefore $y(1) = \frac{3}{2}$. (D)

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7) Evaluate $\int_1^{\sqrt{2}} x\,2^{x^2}\,dx$.

$A)\ \frac{1}{ln(2)}$ $B)\ \frac{1}{2\,ln(2)}$ $C)\ \frac{2}{ln(2)}$ $D)\ \frac{1}{\sqrt{2}\,ln(2)}$ $E)\ \frac{\sqrt{2}}{ln(2)}$ $F)\ \frac{4}{ln(2)}$ $G)\ \frac{2\sqrt{2}}{ln(2)}$
$H)\ \frac{4\sqrt{2}}{ln(2)}$ $H)\ \frac{1}{4\,ln(2)}$ $I)\ \frac{1}{4\sqrt{2}\ ln(2)}$ $J)\ \frac{1}{2\sqrt{2}\ ln(2)}$

solution: $(\ u = x^2,\ du = 2x\,dx) \;=\; \frac{1}{2}\int_1^2 2^u du = \frac{1}{2\,ln(2)}(2^u)\,\Big|_1^2 \;=\; \frac{1}{ln(2)}.$ (A)

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8) Given the differential equation $\frac{dy}{dt} = -\,3y$, with initial condition $y(-\,\frac{1}{3}) = 4\,e$, then find $y(1)$, to 3 decimal places.

$A)$ .134 $B)$ .142 $C)$ .156 $D)$ .169 $E)$ .174 $F)$ .187 $H)$ .199
$I)$ .208 $J)$ .216

solution: $y(t) = y(0)\,e^{-3t}$. $y(-\,\frac{1}{3}) = y(0)\,e^1 = 4e \Rightarrow y(0) = 4,$
and $y(t) = 4\,e^{-3t}$. $y(1) = 4\,e^{-3} \sim$ .199. (H)

9) From the seperable differential equation $\frac{dy}{dx} = 6\sqrt{x\,y}$, $y > 0$, with initial condition $y(1) = 16$, we get the solution :
$A)\ y = (x^2 + 3)^2$ $B)\ y = (2x^2 + 2)^2$ $C)\ y = (3x^2 + 1)^2$ $D)\ y = (x^{3/2} + 3)^2$
$E)\ y = (2x^{3/2} + 2)^2$ $F)\ y = (3x^{3/2} + 1)^2$ $G)\ y = (x^{1/2} + 3)^2$ $H)\ y = (2x^{1/2} + 2)^2$
$I)\ y = (3x^{1/2} + 1)^2$ $J)\ y = 16\,x^2$

<u>solution</u>: By seperation we get, $y^{-\frac{1}{2}}\frac{dy}{dx} = 6x^{\frac{1}{2}} \Rightarrow \int y^{-\frac{1}{2}}dy = 6\int x^{\frac{1}{2}}dx + C \Rightarrow$ $2y^{\frac{1}{2}} = 4x^{\frac{3}{2}} + C$. Using the initial condition we get, $8 = 4 + C \Rightarrow C = 4$. Then $2y^{\frac{1}{2}} = 4x^{\frac{3}{2}} + 4 \Rightarrow \ y = (2x^{\frac{3}{2}} + 2)^2$. (E)

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10) The half life of Polonium-210 is 139 days. Approximately how many days will it take to have 95% of the original sample of Polonium, gone ? (i.e. 5% left.)
$A)$ 300 $B)$ 350 $C)$ 400 $D)$ 450 $E)$ 500 $F)$ 550 $G)$ 600 $H)$ 650 $I)$ 700

<u>solution</u>: Given $\frac{1}{2}y(0) = y(0)\,e^{139k} \Rightarrow k = -\frac{ln(2)}{139} \Rightarrow y(t) = y(0)\,e^{-\frac{ln(2)}{139}t}$.
Now, find t such that $y(t) = \frac{1}{20}y(0) \Rightarrow \frac{1}{20} = e^{-\frac{ln(2)}{139}t} \Rightarrow t = \frac{ln(20)}{ln(2)}139 \sim 600$ (G)

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11) Use the method of integration by parts to evaluate $\int_0^1 arctan(x)\,dx$.
$A)\ \frac{\pi}{4} - \frac{ln(2)}{2}$ $B)\ \frac{\pi}{2} - \frac{ln(3)}{2}$ $C)\ \frac{\pi}{3} - \frac{ln(2)}{4}$ $D)\ \frac{\pi}{6} - \frac{ln(4)}{2}$ $E)\ \frac{\pi}{4} - \frac{ln(3)}{2}$
$F)\frac{\pi}{4} + \frac{ln(2)}{2}$ $G)\ \frac{\pi}{2} + \frac{ln(3)}{2}$ $H)\ \frac{\pi}{3} + \frac{ln(2)}{4}$ $I)\ \frac{\pi}{6} + \frac{ln(4)}{2}$ $J)\ \frac{\pi}{4} + \frac{ln(3)}{2}$

<u>solution</u>: ($u = arctan(x)$, $dv = dx$, $du = \frac{1}{1+x^2}\,dx$, $v = x$)
$\int arctan(x)\,dx = x\,arctan(x) - \int \frac{x}{1+x^2}\,dx$. Using substitution, $w = 1 + x^2$, $dw = 2x\,dx$, we get $\int \frac{x}{1+x^2}\,dx = \frac{1}{2}ln(1 + x^2)$. Therefore
$\int_0^1 arctan(x)\,dx = x\,arctan(x) - \frac{1}{2}ln(1 + x^2)\,\Big|_0^1 = \frac{\pi}{4} - \frac{ln(2)}{2}$. (A)

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12) Evaluate $\int_0^1 x^2\,e^x\,dx$.
$A)$ .624856 $B)$ .684322 $C)$ .718282 $D)$ .752942 $E)$ .787623
$F)$ .812543 $G)$ .865428 $H)$ .918765 $I)$ .965422 $J)$ .985622

<u>solution</u>: $(u = x^2,\ dv = e^x dx,\ du = 2x,\ v = e^x)$. $\int x^2\,e^x\,dx =$ $x^2\,e^x - 2\int x\,e^x dx$. Next, using $u = x,\ dv = e^x dx,\ du = dx,\ v = e^x$, on $\int x\,e^x dx$, we get $\int x^2\,e^x\,dx = x^2\,e^x - 2(x\,e^x - \int e^x\,dx$. Then
$\int_0^1 x^2\,e^x\,dx = x^2\,e^x - 2x\,e^x + 2\,e^x\,\Big|_0^1 = e - 2 \sim .718282$ (C)