

MATH 311

Topics in Applied Mathematics

Lecture 7:
Vector spaces.
Subspaces.

Linear operations on vectors

Let $\mathbf{x} = (x_1, x_2, \dots, x_n)$ and $\mathbf{y} = (y_1, y_2, \dots, y_n)$ be n -dimensional vectors, and $r \in \mathbb{R}$ be a scalar.

Vector sum: $\mathbf{x} + \mathbf{y} = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n)$

Scalar multiple: $r\mathbf{x} = (rx_1, rx_2, \dots, rx_n)$

Zero vector: $\mathbf{0} = (0, 0, \dots, 0)$

Negative of a vector: $-\mathbf{y} = (-y_1, -y_2, \dots, -y_n)$

Vector difference:

$$\mathbf{x} - \mathbf{y} = \mathbf{x} + (-\mathbf{y}) = (x_1 - y_1, x_2 - y_2, \dots, x_n - y_n)$$

Properties of linear operations

$$\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$$

$$(\mathbf{x} + \mathbf{y}) + \mathbf{z} = \mathbf{x} + (\mathbf{y} + \mathbf{z})$$

$$\mathbf{x} + \mathbf{0} = \mathbf{0} + \mathbf{x} = \mathbf{x}$$

$$\mathbf{x} + (-\mathbf{x}) = (-\mathbf{x}) + \mathbf{x} = \mathbf{0}$$

$$r(\mathbf{x} + \mathbf{y}) = r\mathbf{x} + r\mathbf{y}$$

$$(r + s)\mathbf{x} = r\mathbf{x} + s\mathbf{x}$$

$$(rs)\mathbf{x} = r(s\mathbf{x})$$

$$1\mathbf{x} = \mathbf{x}$$

$$0\mathbf{x} = \mathbf{0}$$

$$(-1)\mathbf{x} = -\mathbf{x}$$