

Lecture on Rules of Inference

Discuss example: what do you know from the following facts?

1. X and Y both need 87 in final to reach 450 pts.
2. X got 92 in final.
3. Y didn't get A.

Three basic rules

- **Modus Ponens** $(p, p \rightarrow q) \implies \therefore q$.
- **Modus Tollens** $(\neg q, p \rightarrow q) \implies \therefore \neg p$.
- **Hypothetical Syllogism** $(p \rightarrow q), (q \rightarrow r) \implies \therefore p \rightarrow r$.

Rule of inference: to show $p \rightarrow q$ is true, only need to consider the case when p is true. Other easy rules:

- **Conjunction** $(p, q) \implies p \wedge q, (p \wedge q) \implies p$.
- **Disjunction** $p \implies p \vee q, (p \vee q, \neg p) \implies \therefore q$.
- **Resolution** $(p \vee q, \neg p \vee r) \implies \therefore q \vee r$.

Also any logical equivalence

Example 1. Build an argument for $(p, p \rightarrow \neg q, \neg q \rightarrow \neg r) \implies \therefore \neg r$.

Example 2. Build an argument for $p \rightarrow r, r \rightarrow s, t \vee \neg s, \neg t \vee u, \neg u$ then $\neg p$.

A trick: Show the logic equivalence $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$.

To show that $p \implies \therefore (q \rightarrow r)$, we can do $p, q \implies \therefore r$.

Example 3. Build an argument for $u \rightarrow r, (r \wedge s) \rightarrow (p \vee t), q \rightarrow (u \wedge s)$ and $\neg t$ imply $q \rightarrow p$.

How about $p, p \vee q, q \rightarrow (r \rightarrow s)$ and $t \rightarrow r$ imply $\neg s \rightarrow \neg t$?

– How to show “invalid”?