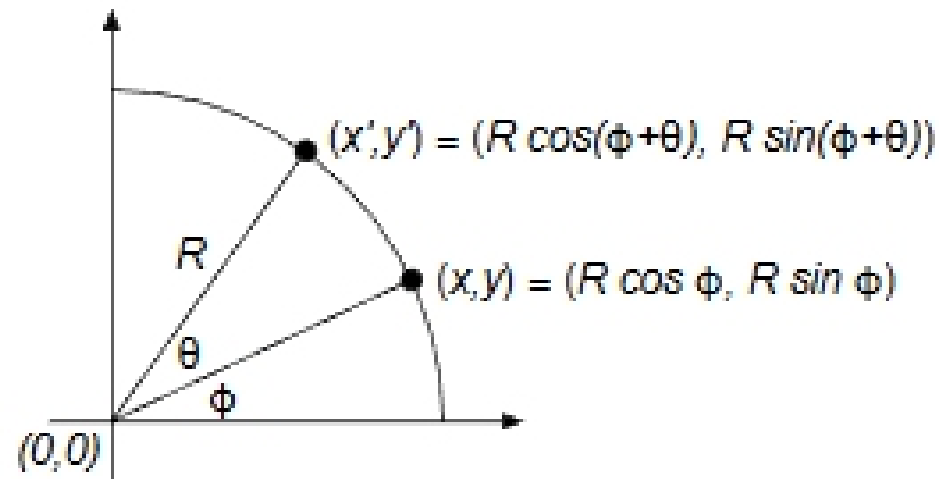


2D Transformations and Homogeneous Coordinates

CS 442/542

September 8, 2011

2D CCW rotation about origin



$$\begin{aligned}x' &= R \cos(\phi + \theta) \\ &= R(\cos \phi \cos \theta - \sin \phi \sin \theta) \quad (\text{trig identity}) \\ &= x \cos \theta - y \sin \theta \\ y' &= R \sin(\phi + \theta) \\ &= R(\sin \phi \cos \theta + \cos \phi \sin \theta) \quad (\text{trig identity}) \\ &= y \cos \theta + x \sin \theta\end{aligned}$$

Matrix representation of rotation

- 1 × 2 row vector times a 2 × 2 matrix

$$[x' \ y'] = [x \ y] \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

- 2 × 2 matrix times a 2 × 1 column vector (our convention)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- *Orthonormal Matrix* represents *rigid transformation*
 - Rows orthogonal, columns orthogonal
 - Each row and column has unit magnitude