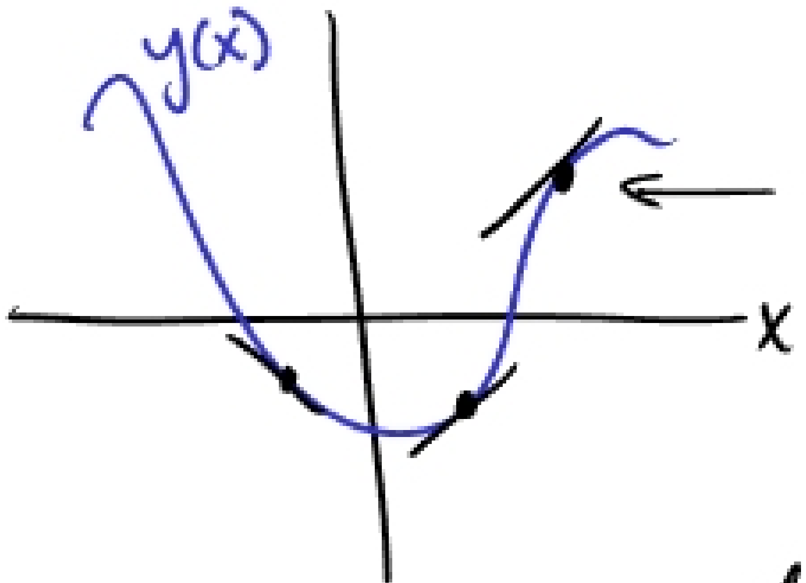
Slope fields and Solution curves
(Geometric approach to DEs)

lets consider a first-order Differential equation

$\frac{dy}{dx} = f(x,y)$ e.g. $\frac{dy}{dx} = y^2$, $\frac{dy}{dx} = xy$

Can't directly integrate because right hand side depends on y, the unknown function.

What does the equation mean geometrically

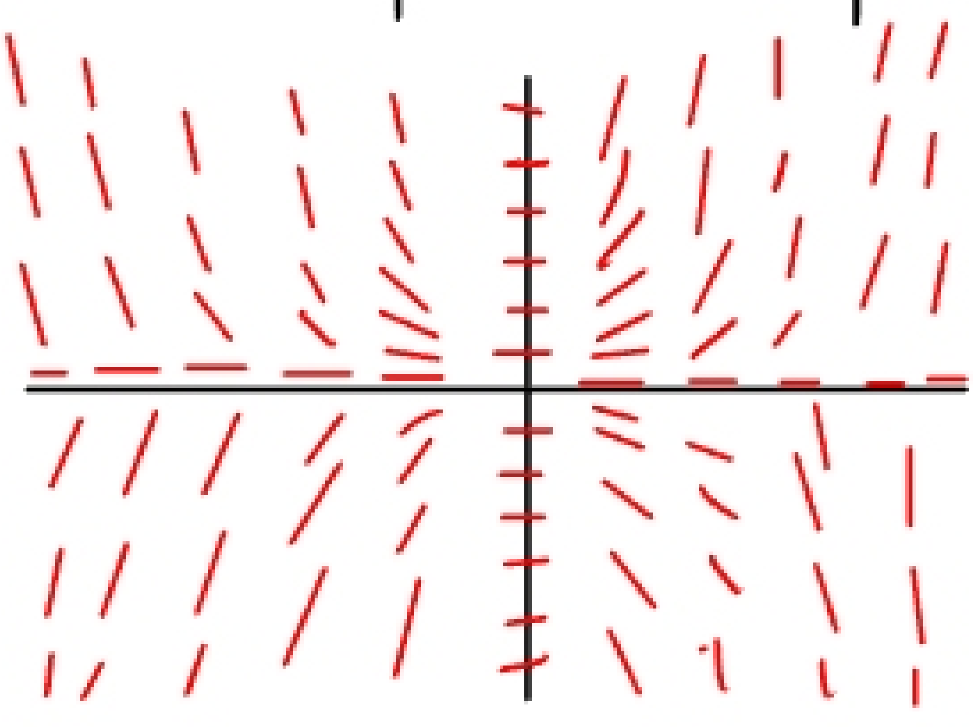


$\longleftarrow$ slope $= f(x,y(x))$

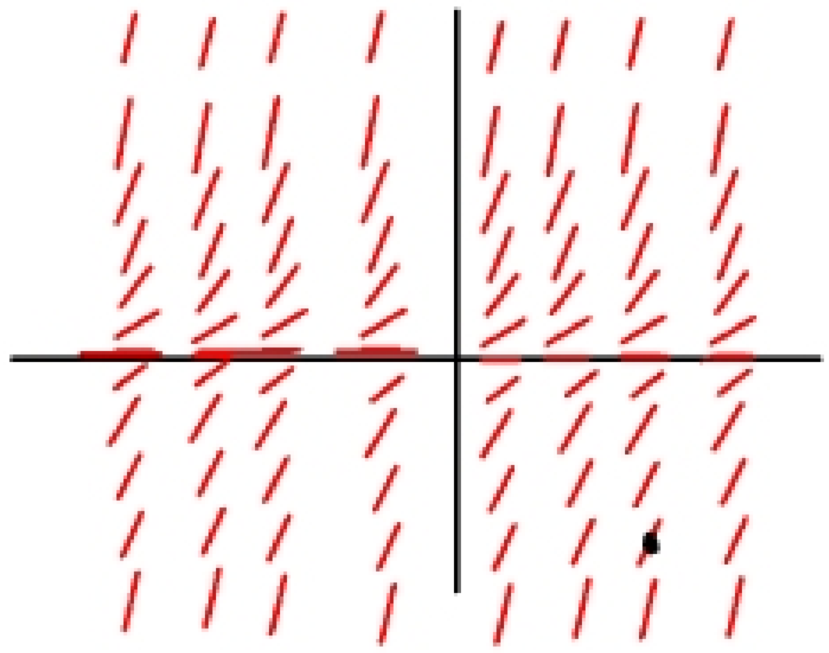
That is, the solution curve $y(x)$ (whatever it may be) must have at each point a slope $f(x,y(x))$ depending on x and the value of the solution $y(x)$, that is, depending on where we are in the xy-plane

<u>Idea</u>: Plot all the possible slopes $\Longrightarrow$ get <u>Slope field</u>

$f(x,y) = xy$



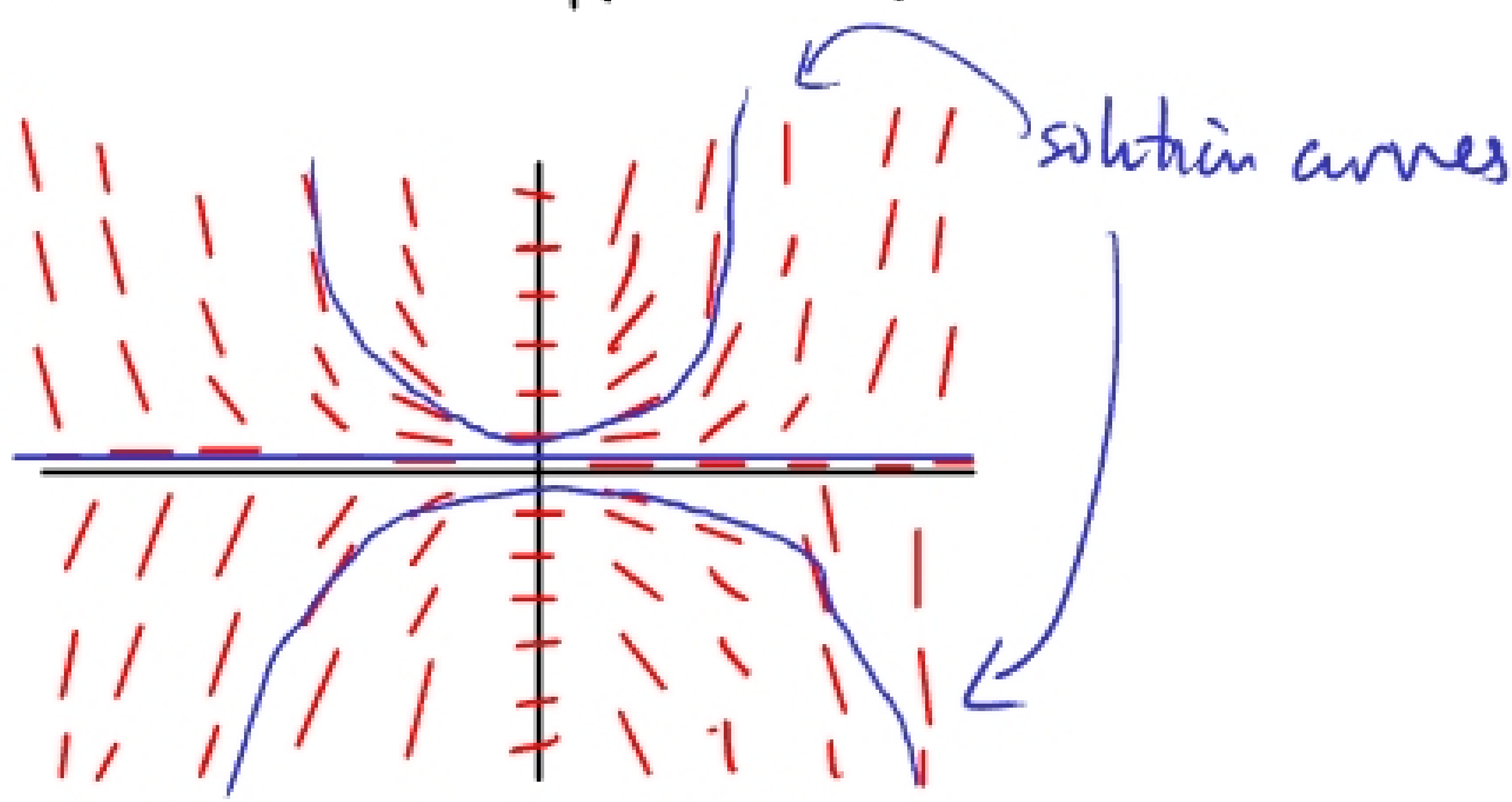
$f(x,y) = y^2$



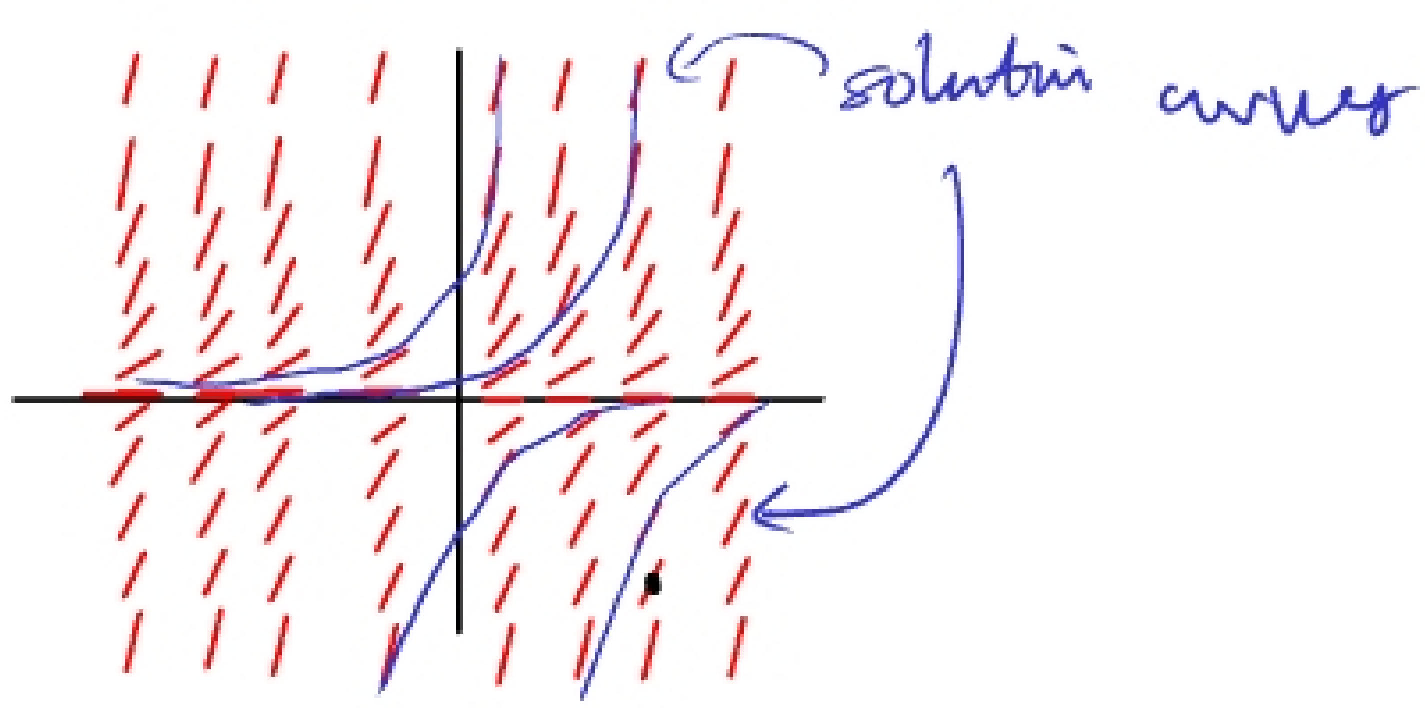
The marking means that <u>if</u> the solution passes through a particular point then <u>it must</u> have the prescribed slope. (but we don't know at the start where the solution goes.

We can use slope field to approximately sketch solutions

$f(x,y) = xy$



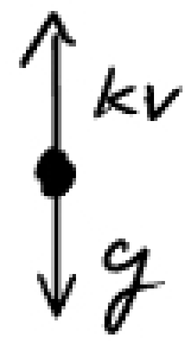
$f(x,y) = y^2$



Slope fields are useful to get Qualitative info about solutions:

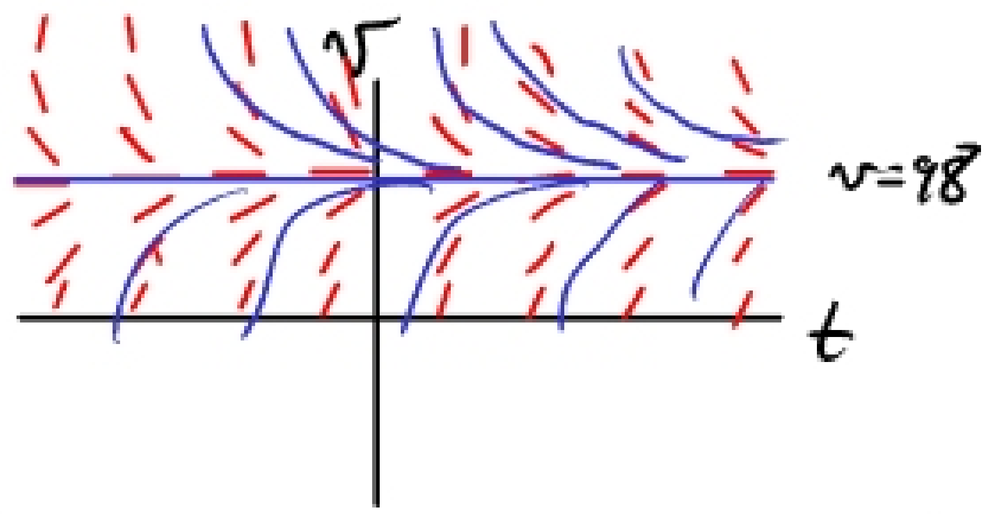
<u>Example</u> Object falling with air resistance let's take down ward velocity to be positive. The simplest model is when air resistance is proportional to velocity

$$a = \frac{dv}{dt} = g - kv$$



Lets say $g = 9.8\ m/s^2$ and $k = .1\ s^{-1}$

The equation is $\frac{dv}{dt} = f(t,v)$ so plot slope field in (t,v)



$f = 0$ if $v = g/k = 98$

All solution curves asymptote to the constant solution $v = 98\ m/s$ ⟵ called terminal velocity.

More generally, we can consider

AUTONOMOUS FIRST ORDER Diff. Eq. $\frac{dy}{dx} = f(y)$ (no x on RHS)

Then the slope field only veries in the veritical direction