OSCILLATIONS ABOUT EQUILIBRIUM

Some definitions for periodic motion:

Period: $T \equiv$ time required for one cycle of a periodic motion

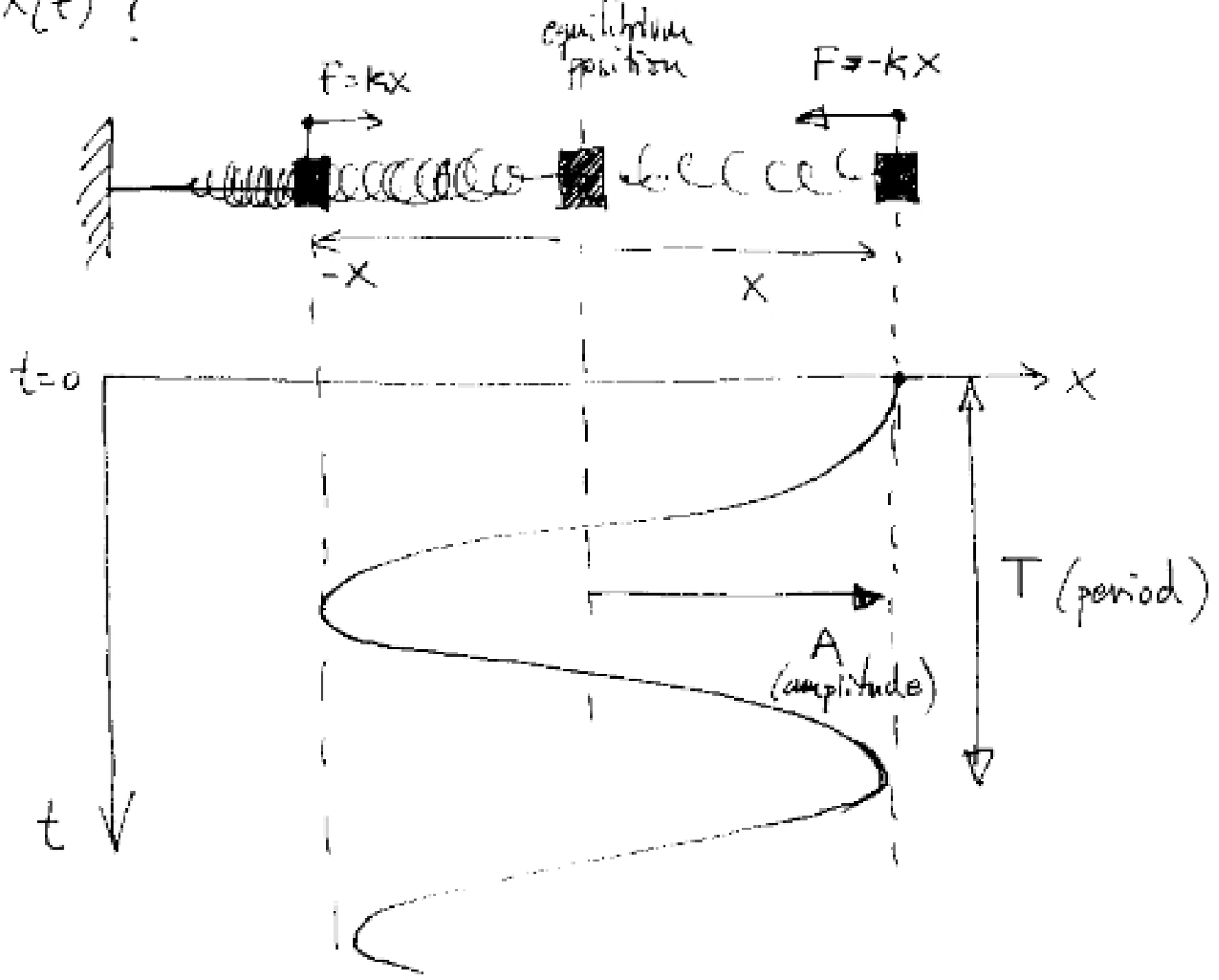
Frequency: $f = \frac{1}{T}$ $\quad [f] = \frac{1}{s} = Hz$ $\quad 1\,Hz = 1$ cycle/second

Circular periodic motion:

Frequency (angular): $\omega = 2\pi f$ $\quad [\omega] \equiv \frac{rad}{second} = \frac{1}{s} = Hz/2\pi$

How to describe the periodic motion of an object?

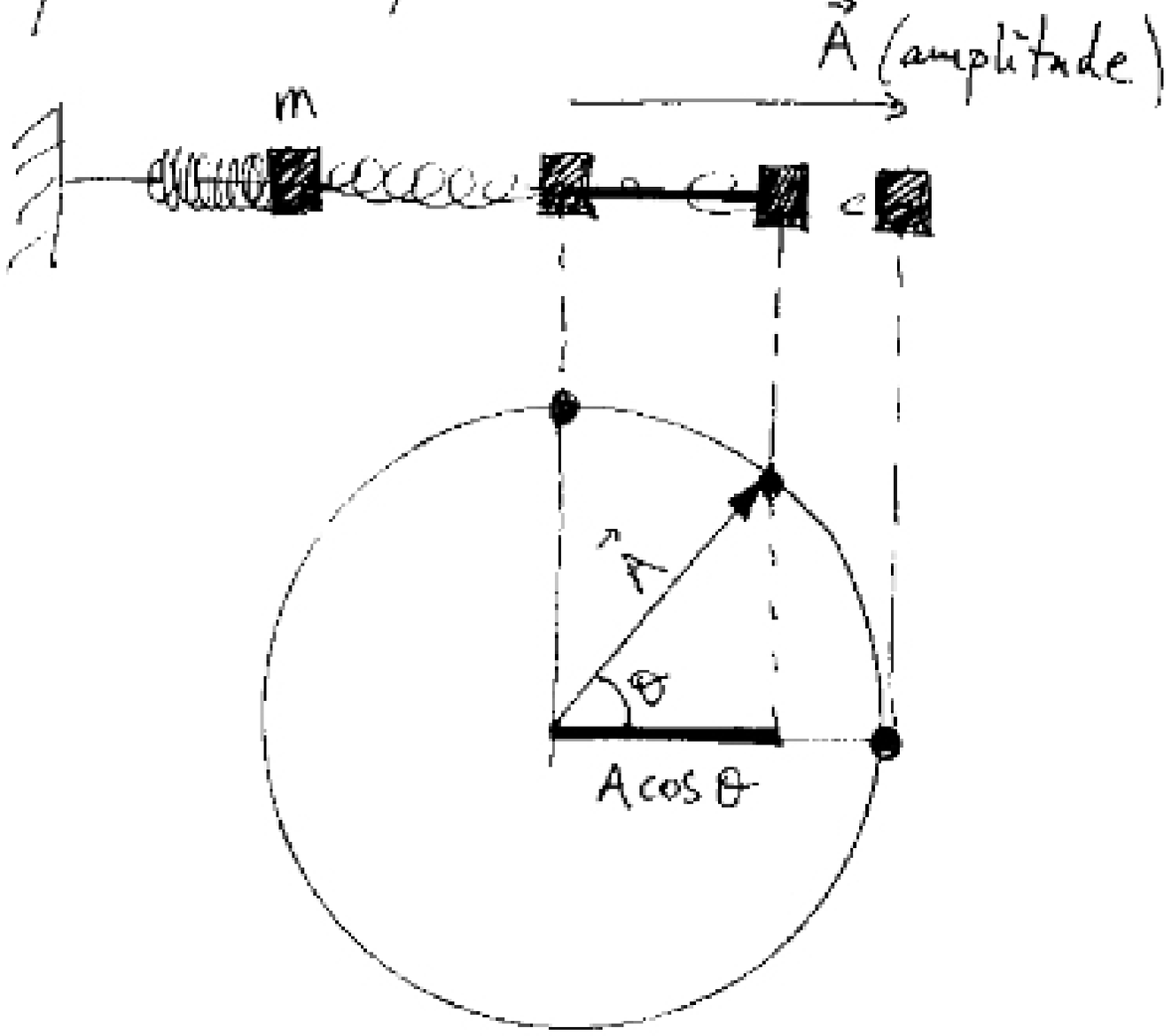
$x(t)$?



What's the expression of "x" as a function of time?

→ We need calculus to find it.

Hint: We can use what we know about circular motion to find the expression.



This means that the position of the mass "m" is given by—

$$x = A\cos\theta$$

We know that in rotational motion, $\theta = \omega t$, so:

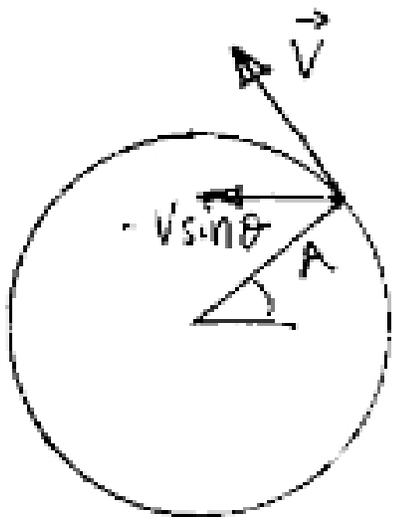
$$\boxed{x = A\cos(\omega t) = A\cos(2\pi f t) = A\cos\left(\frac{2\pi}{T}t\right)}$$

* 1st equation of periodic motion for position.

$x_{MAX} = \pm A$ $\quad \left[\cos(\omega t) = \pm 1 \Rightarrow \omega t = n\pi \Rightarrow t = \frac{n\pi}{\omega} = \frac{n\pi}{2\pi}T = \frac{n}{2}T\right]$

$x_{MIN} = 0$ $\quad \left[\cos(\omega t) = 0 \Rightarrow \omega t = (2n+1)\frac{\pi}{2} \Rightarrow t = n\frac{T}{4}\right]$

What about the velocity ?



$$V_x = -V\sin\theta = -A\omega\sin(\omega t)$$

$V = r\omega$

$$\boxed{V = -A\omega\sin(\omega t)}$$

$V_{MAX} = -A\omega \quad [\sin(\omega t) = \pm 1]$

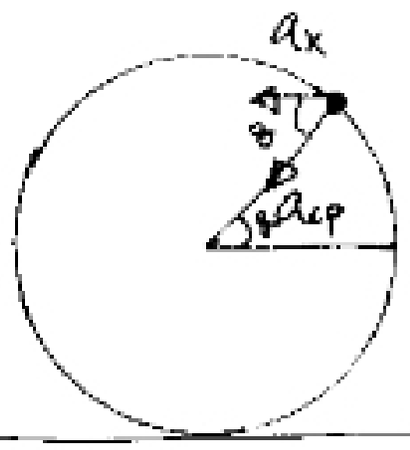
2nd. equation of periodic motion for velocity.

V_{MAX} : $[\sin(\omega t) = \pm 1 \Rightarrow \omega t = (2n+1)\frac{\pi}{2} \Rightarrow t = \frac{(2n+1)}{\omega}\frac{\pi}{2}$

$\Rightarrow t = (2n+1)\frac{\pi}{2}\frac{T}{2\pi} = (2n+1)\frac{T}{4}$

$V_{MIN} = 0 \quad [\sin(\omega t) = 0 \Rightarrow \omega t = n\pi \Rightarrow t = n\frac{T}{2}]$

What about the acceleration ?



$$a_{cp} = r\omega^2$$

$$a_x = -a_{cp}\cos\theta = -r\omega^2\cos(\omega t)$$

$$\boxed{a = -A\omega^2\cos(\omega t)}$$

$a_{MAX} = -A\omega^2 \quad [\cos(\omega t) = \pm 1, \; t = n\frac{T}{2}]$

$a_{MIN} = 0 \quad [\cos(\omega t) = 0, \; t = n\frac{T}{4}]$