ME451: Control Systems

Lecture 21
Root locus: Lag compensator &
Lead-lag compensator design

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Course roadmap

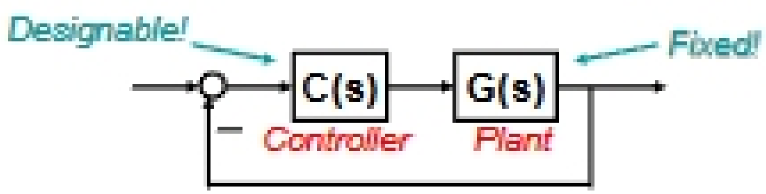
Modeling
- ✔ Laplace transform
- ✔ Transfer function
- ✔ Models for systems
  - ✔ electrical
  - ✔ mechanical
  - ✔ electromechanical
- ✔ Block diagrams
- ✔ Linearization

Analysis
- ✔ Time response
  - ✔ Transient
  - ✔ Steady state
- Frequency response
  - Bode plot
- ✔ Stability
  - ✔ Routh-Hurwitz
  - Nyquist

Design
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

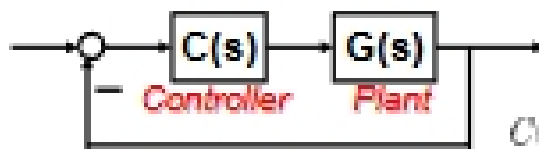
Closed-loop design by root locus



- Place closed-loop poles at desired location
  - by tuning the gain C(s)=K.
- If root locus does not pass the desired location, then reshape the root locus
  - by adding poles/zeros to C(s).

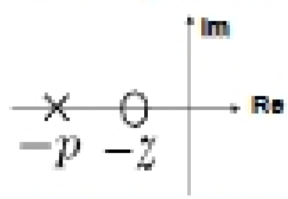
Compensation

Lead and lag compensators (review)

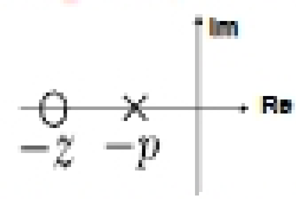


$$C(s) = K\frac{s+z}{s+p}, \ (z>0, p>0)$$

- **Lead compensator**



- **Lag compensator**

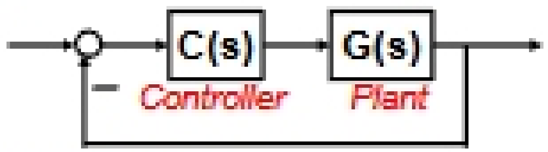


The reason why these are called "lead" and "lag" will be explained in frequency response approach (later in this course).

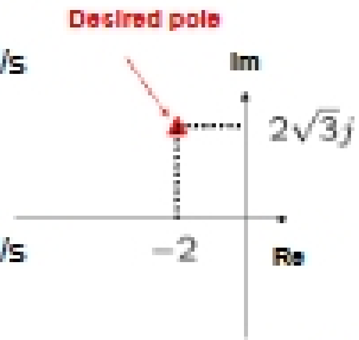
Lead-lag compensator design

- Consider a system

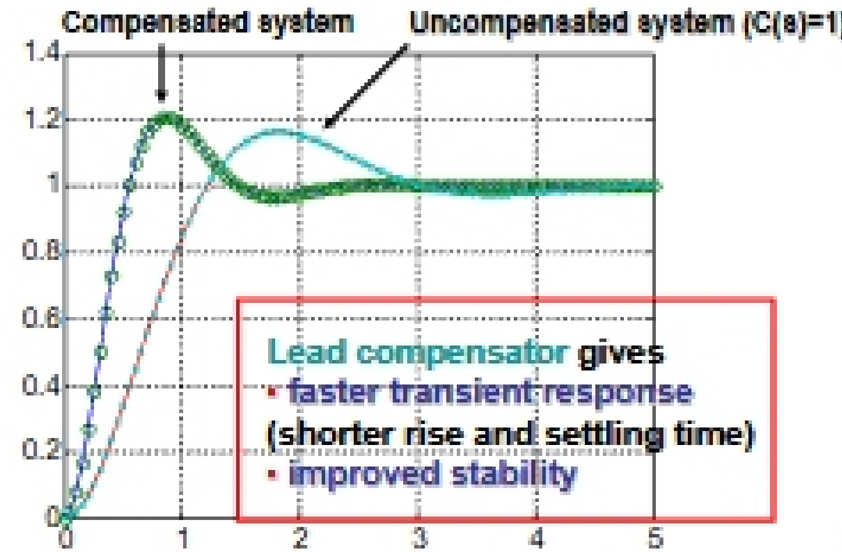
$$G(s) = \frac{4}{s(s+2)}$$



- Analysis of CL system for C(s)=1
  - Damping ratio ζ=0.5
  - Undamped natural freq. ω_n=2 rad/s
  - Ramp-error constant Kv=2
- Performance specification
  - Damping ratio ζ=0.5
  - Undamped natural freq. ω_n=4 rad/s
  - Ramp-error constant Kv=50



Comparison of step responses (after lead compensation)



Error constants (after lead compensation)

$$G(s)C_{Lead}(s) = \frac{4}{s(s+2)} \cdot \frac{4.675(s+2.9)}{s+5.4}$$

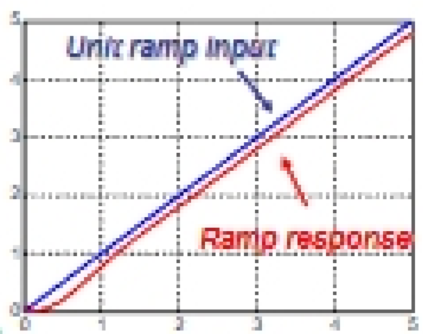
- Step-error constant

$$K_p := \lim_{s\to 0} G(s)C_{lead}(s) = \infty$$

- Ramp-error constant

$$K_v := \lim_{s\to 0} sG(s)C_{lead}(s) = 5.02$$

NOT SATISFACTORY!



Lag compensator can reduce steady-state error.

How to design lag compensator

- Lag compensator $C_{Lag}(s) = \frac{s+z}{s+p}$
- We want to increase ramp-error constant

$$K_v = \lim_{s\to 0} sG(s)C_{Lead}(s)C_{Lag}(s) = 5.02 \cdot \frac{z}{p} > 50$$

Take, for example, z=10p.

- We do not want to change CL pole location s_1 so much (already satisfactory transient).

$$\left.\begin{array}{r} 1+G(s_1)C_{Lead}(s_1)=0 \\ C_{Lag}(s_1)\approx 1 \end{array}\right\} \Rightarrow 1+G(s_1)C_{Lead}(s_1)C_{Lag}(s_1)\approx 0$$