

MATH 311-504

Topics in Applied Mathematics

Lecture 2-2:

**Linear maps (continued).
Matrix transformations.**

Vector space

Vector space is a set V equipped with two operations $\alpha : V \times V \rightarrow V$ and $\mu : \mathbb{R} \times V \rightarrow V$ that have certain properties (listed below).

The operation α is called *addition*. For any $\mathbf{u}, \mathbf{v} \in V$, the element $\alpha(\mathbf{u}, \mathbf{v})$ is denoted $\mathbf{u} + \mathbf{v}$.

The operation μ is called *scalar multiplication*. For any $r \in \mathbb{R}$ and $\mathbf{u} \in V$, the element $\mu(r, \mathbf{u})$ is denoted $r\mathbf{u}$.

Properties of addition and scalar multiplication

- A1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$ for all $\mathbf{a}, \mathbf{b} \in V$.
- A2. $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$ for all $\mathbf{a}, \mathbf{b}, \mathbf{c} \in V$.
- A3. There exists an element of V , called the *zero vector* and denoted $\mathbf{0}$, such that $\mathbf{a} + \mathbf{0} = \mathbf{0} + \mathbf{a} = \mathbf{a}$ for all $\mathbf{a} \in V$.
- A4. For any $\mathbf{a} \in V$ there exists an element of V , denoted $-\mathbf{a}$, such that $\mathbf{a} + (-\mathbf{a}) = (-\mathbf{a}) + \mathbf{a} = \mathbf{0}$.
- A5. $r(\mathbf{a} + \mathbf{b}) = r\mathbf{a} + r\mathbf{b}$ for all $r \in \mathbb{R}$ and $\mathbf{a}, \mathbf{b} \in V$.
- A6. $(r + s)\mathbf{a} = r\mathbf{a} + s\mathbf{a}$ for all $r, s \in \mathbb{R}$ and $\mathbf{a} \in V$.
- A7. $(rs)\mathbf{a} = r(s\mathbf{a})$ for all $r, s \in \mathbb{R}$ and $\mathbf{a} \in V$.
- A8. $1\mathbf{a} = \mathbf{a}$ for all $\mathbf{a} \in V$.