

Lecture-17

Computing Optical Flow: Lucas & Kanade Global Flow

Lucas & Kanade (Least Squares)

- Optical flow eq

$$f_x u + f_y v = -f_t$$

- Consider 3 by 3 window

$$f_{x1} u + f_{y1} v = -f_{t1}$$

⋮

$$f_{x9} u + f_{y9} v = -f_{t9}$$

$$\begin{bmatrix} f_{x1} & f_{y1} \\ \vdots & \vdots \\ f_{x9} & f_{y9} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -f_{t1} \\ \vdots \\ -f_{t9} \end{bmatrix}$$

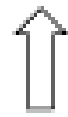
$$\mathbf{A} \mathbf{u} = \mathbf{f}_t$$

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$$\mathbf{A}\mathbf{u} = \mathbf{f}_t$$

$$\mathbf{A}^T \mathbf{A}\mathbf{u} = \mathbf{A}^T \mathbf{f}_t$$

$$\mathbf{u} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{f}_t$$



$$\min \sum (f_{xi}u + f_{yi}v + f_t)^2$$

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$$\min \sum (f_{xi}u + f_{yi}v + f_t)^2$$



$$\sum (f_{xi}u + f_{yi}v + f_t)f_{xi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_t)f_{yi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ni})f_{xi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ni})f_{yi} = 0$$

$$\sum f_{xi}^2 u + \sum f_{xi}f_{yi}v = -\sum f_{xi}f_{ni}$$

$$\sum f_{xi}f_{yi}u + \sum f_{yi}^2 v = -\sum f_{yi}f_{ni}$$

$$\begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi}f_{yi} \\ \sum f_{xi}f_{yi} & \sum f_{yi}^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum f_{xi}f_{ni} \\ -\sum f_{yi}f_{ni} \end{bmatrix}$$

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$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi}f_{yi} \\ \sum f_{xi}f_{yi} & \sum f_{yi}^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum f_{xi}f_{ni} \\ -\sum f_{yi}f_{ni} \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi}f_{yi})^2} \begin{bmatrix} \sum f_{xi}^2 & -\sum f_{xi}f_{yi} \\ -\sum f_{xi}f_{yi} & \sum f_{yi}^2 \end{bmatrix} \begin{bmatrix} -\sum f_{xi}f_{ni} \\ -\sum f_{yi}f_{ni} \end{bmatrix}$$

$$u = \frac{-\sum f_{xi}^2 \sum f_{yi}f_{ni} + \sum f_{xi}f_{yi} \sum f_{xi}f_{ni}}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi}f_{yi})^2}$$

$$v = \frac{\sum f_{xi}f_{yi} \sum f_{xi}f_{ni} - \sum f_{xi}^2 \sum f_{yi}f_{ni}}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi}f_{yi})^2}$$