

LESSON #4: VALIDITY & SOUNDNESS (2.1) / METHODS OF PROOF (2.2) / FORMAL PROOFS (2.3)

Section 2.1: Valid and sound arguments

PowerPoint #1

A **valid** argument is one where the conclusion is a **logical consequence** of its premises. This happens when the form of the argument is such that it would be *impossible* for the conclusion to be false as long as the premises are true. Or, stated differently, *if* the premises are true, then the conclusion *must* be true.

Which of the following two arguments is **invalid**, and how can you know???

Socrates is a man.
All men are mortal.
Socrates is mortal. [The conclusion is a logical consequence of the premises = **valid** argument]

Lucretius is mortal.
All men are mortal.
Lucretius is a man. [The conclusion is **NOT** a logical consequence of the premises. What if Lucretius is my mortal *goldfish*. Then the premises would be true in that situation (world) but the *conclusion* would be *false*. So, this is an **invalid** argument.]

PowerPoint #2

Notice from this that an **invalid** argument (like the one just considered above) can have *true premises*.

PowerPoint #3

Indeed, an **invalid** argument may be built on not only **true premises** but a **true conclusion** as well. That is, for an argument to be valid, it's not enough that the premises and conclusion all be true; the *premises and conclusion must be related to each other* in such a way that the conclusion is a logical consequence of the premises. Consider:

The moon revolves around the earth.
The earth revolves around the sun.
Therefore, Mars revolves around the sun.

All three of the above statements are true in our world, and yet the last statement (the conclusion) is **NOT** a logical consequence of the other two statements. Therefore, this argument is **NOT** valid. There is nothing about the truth of the first two statements that makes it necessary that the third statement (the conclusion) be true. Indeed, one can imagine a universe (a 'world', as we've been using the term) in which the name 'Mars' labels a planet in some *other* solar system around a different star. In that case,

the premises in the above argument would be true but the conclusion would be false (assuming 'the sun' labels our own star). Since this is possible, the argument is not valid.

PowerPoint #4

Conversely, an argument may be *valid* not only when one or more of its *premises* is false in some particular world *but also even if the conclusion is false* in that world. This is because **validity** is determined only by the **form/structure** of the argument, **not** the truth values of its statements in any particular world. Consider the following argument:

The moon is made entirely of blue cheese.
Blue cheese smells bad.
Therefore, the moon smells bad.

In our real world, the first premise above and the conclusion are both false, **but the argument is still valid because of its form**. That is, *if* it were indeed true that the moon is made of blue cheese and such cheese smells bad, then it would necessarily be the case that the moon smells bad.

PowerPoint #5

Even more to the extreme, it is possible to think of **valid** arguments that don't have a single true statement in them (i.e., true as evaluated in regard to some particular world). Consider the following argument relative again to the real world we all live in:

The moon is made entirely of blue cheese.
Blue cheese always glows pink.
Therefore, the moon glows pink.

Every line of the above argument is false in our world, and yet the argument is **valid** because the conclusion is a logical consequence of the two premises (i.e., *if* the premises were both true, then the conclusion would necessarily be true as well, because of the way the sentences are related to each other). Again, this shows that validity isn't directly related to truth in any particular world and that you can determine whether an argument is valid or not without even knowing the truth-values of its premises and conclusion in any particular world but just by looking at the form of the argument itself.

PowerPoint #6

Of course, we normally want our arguments to be not only *valid* in form but *true* as well; that is, having both true premises and a true conclusion. When this is the case for an argument relative to a particular world, we say that the argument is not only valid but **sound** for that world.

(Note that an *invalid* argument is automatically *unsound in all worlds*, regardless whether the premises and conclusion may be true in some worlds.)

So, whereas a *valid* argument ensures the *logical consequence* of its conclusion, a *sound* argument further ensures the *truth* of its conclusion in a particular world.

PowerPoint #7

Notice that **validity** is a feature of an argument itself **without** regard for particular worlds. (Another way to say this is that an argument is simply either valid or invalid for *all* worlds.) **Soundness**, in contrast, is always **relative to a world**. The same valid argument may be sound in one world but unsound in another.

That said, logic normally deals only with the validity of arguments, not their soundness (because the truth of premises is something the logician leaves up to experts in other fields (e.g., historians, psychologists, physicists, etc.).

Powerpoint #8

Open Tarski's World, Socrates' Sentences, and Socrates' World (from Exercise 2.1 on p. 44 of textbook)

Evaluate several of the arguments in Socrates' Sentences as valid or invalid, then as sound or unsound in Socrates' world ...

PowerPoint slide #9

More practice evaluating validity and soundness:

Is the argument in the powerpoint slide **valid**? (Note that 'P1' means 'premise 1', 'P2' means 'premise 2', etc., and 'C' means 'conclusion')

Is the argument **sound** in Hamilton's Bizarre Little World?

Answers:

No, the argument is **not valid**, and therefore we know automatically that it is **not sound** either.

The fact that all of the premises and the conclusion are true in Hamilton's Bizarre Little World does *not* indicate that the argument here is valid. Remember that an *invalid* argument can sometimes be made up of *all true* statements (i.e., all true in some particular world), just as a *valid* argument can sometimes be made up of *all false* statements (i.e., all false in some particular world). The real question here is not whether all the statements are true in the world, but **whether the conclusion is a logical consequence of the premises**. It is **not**, because the truth of the premises does not guarantee that this conclusion will also be true. We can see this by simply imagining a world in which objects 'a' and 'c' switch places on the grid. In that counterexample world, all three premises of the argument would still be true, but the conclusion would now be *false*. This shows that the conclusion of this argument is not a logical consequence of the premises.