LESSON 9: AMBIGUITY, EQUIVALENCES, TRANSLATION (3.5 -3.7)

Reading assignment pp. 79-86

POWERPOINT SLIDE #1

Parentheses are used to disambiguate FOL sentences, such as where **a combination of *disjunctions* and *conjunctions*** otherwise makes it unclear how to interpret the sentence:

Bad(bob) ∨ Bewildered(bob) ∧ Bogeyman(bob) **????** (*ambiguous*)

(Bad(bob) ∨ Bewildered(bob)) ∧ Bogeyman(bob)
*"Bob is bad or bewildered, **and** he is a bogeyman."*

*If we group the **first two** predicates together and make the conjunction the main connective of the sentence, then Bob is definitely a bogeyman . . .*

Bad(bob) **∨** (Bewildered(bob) ∧ Bogeyman(bob))
*"Bob is bad, **or** he's a bewildered bogeyman"*

*If we group the **last two** predicates together and make the disjunction the main connective of the sentence, then Bob is not necessarily a bogeyman . . .*

Note that our textbook doesn't normally require you to add extra parentheses to a sentence containing **only** a series or conjunctions or **only** a series of disjunctions.

POWERPOINT SLIDE #2

Another case in which parentheses can disambiguate a sentence is in some complex sentences containing negation:

¬Bad(bob) ∧ ¬Bogeyman(bob)	*"Bob is not bad, and he's not a bogeyman"*
¬(Bad(bob) ∧ Bogeyman(bob))	*"Bob is not a bad bogeyman"* or *"It's not the case that Bob is both bad and a bogeyman"* (though he might be *just* bad, or *just* a bogeyman)
¬(¬Bad(bob) ∧ ¬Bogeyman(bob))	*"It's not true that Bob is neither bad nor a bogeyman"* (he might bad or a bogeyman or both)

POWERPOINT SLIDE #3

The book emphasizes that in FOL there is sometimes *more than one way to logically state the same thing*. The most important example of such equivalence between sentences of FOL is what is called the **'DeMorgan's Laws.'** (Discuss, focusing on the *meaning* of conjunction and disjunction in each case. The

double-sided arrow indicates that the FOL sentence to its left and the FOL sentence to its right are equivalent, or have the same truth conditions .... This means that the two sentences are true in all the same worlds and false in all the same worlds.)

$$\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$$

$$\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$$

You need to *understand* and (only then) *memorize* the DeMorgan's equivalences. Period.
(Note that the outer parentheses in each of the sentences on the *right* side of the equivalence arrows are optional, but the parentheses in the lefthand sentences are necessary, because of the negation symbol.)

POWERPOINT SLIDE #4

Regarding **translation**, the important points made in this chapter of the textbook are:

- The *minimum* requirement of an FOL translation of an English sentence is that the two sentences **have the same truth values in all possible circumstances** (all possible worlds).
- The *stylistic goal* of an FOL translation of an English sentence is that the FOL **mirror the English**, at least closely enough that a reader could reasonably 'reverse-engineer' your FOL back to the English expression from which you originally translated it.

"It is not true that Claire and Max are both at home"

$\neg(\text{Home}(\text{claire}) \wedge \text{Home}(\text{max}))$
$\neg\text{Home}(\text{claire}) \vee \neg\text{Home}(\text{max})$

The second sentence is the DeMorgan's equivalent of the first and is therefore true in all the same circumstances (i.e., the two sentences have the same 'truth conditions'); however, stylistically the second sentence is farther from the English and so **not** as good of a translation as the first FOL sentence . . .

POWERPOINT SLIDE #5

- The English words *but*, *however*, *yet*, and *nonetheless* are, for purposes of FOL, stylistic variants of *and*. Unlike 'and', they all carry a sense of 'unexpectedness' about what follows, but this extra meaning is irrelevant to the sort of logical analysis we are conducting and can safely be ignored for our purposes. So, the correct translation of **"Bob is boring but he's not stupid"** would be:

$\text{Boring}(\text{bob}) \wedge \neg\text{Stupid}(\text{bob})$

POWERPOINT SLIDE #6

In the instructions to **Exercise 3.21**, the book mentions *a useful technique for checking the accuracy of your translations from English into the blocks language of FOL*: **Build several blocks worlds** that make the English sentence(s) true or false, then check to see if your FOL translations of these sentences have the same truth values (as their English counterparts) in the same worlds (they should).

POWERPOINT SLIDE #7

** **Do the first 6 sentences in Exercise 3.21**, taking suggestions from the class as to how to translate each sentence.

1. *Either **a** is small or both **c** and **d** are large.*
2. ***d** and **e** are both in back of **b**.*
3. ***d** and **e** are both in back of **b** and larger than it.*
4. *Both **d** and **c** are cubes, however neither of them is small.*
5. *Neither **e** nor **a** is to the right of **c** and to the left of **b**.*
6. *Either **e** is not large or it is in back of **a**.*

Answers:

1. Small(a) ∨ (Large(c) ∧ Large(d))
2. BackOf(d, b) ∧ BackOf(e, b)
3. BackOf(d, b) ∧ BackOf(e, b) ∧ Larger(d, b) ∧ Larger(e, b)
4. Cube(d) ∧ Cube(c) ∧ ¬Small(d) ∧ ¬Small(c)
5. ¬(RightOf(e, c) ∧ LeftOf(e, b)) ∧ ¬(RightOf(a, c) ∧ LeftOf(a, b))
6. ¬Large(e) ∨ BackOf(e, a)

POWERPOINT SLIDE #8

Then **check the sentences against Wittgenstein's World** (they should **all be *true***), correcting any translations as necessary to make them true.

POWERPOINT SLIDE #9

Finally, **make the following modifications to Wittgenstein's World** as instructed in the middle paragraph of Exercise 3.22:

> **Make all the large or medium objects small, and the small objects large**. With these changes in the world, the English sentences 1, 3, 4, and 10 become **false**, while the rest remain true. Verify that the same holds for your translations. If not, correct your translations.

POWERPOINT SLIDE #10