LESSON #6: Fitch proofs (2.4); Demonstrating nonconsequence through counterexamples (2.5)

Assigned reading pp. 58-65 (2.4 and 2.5)

PowerPoint slide #1

We don't learn any new formal rules in the remainder of this chapter; instead, we use the rules and concepts we've already learned as we begin using the computer program that *allows us to construct formal proofs*, **Fitch**.

It is important that you follow the "You try it" sections carefully, step-by-step, and refer to the software manual on the Fitch program as needed, to make sure that you will know how to open, construct, and save proofs in Fitch.

Probably the best way I can help you here is to do some problems with you.

PowerPoint slide #2

For **Exercise 2.5** you had to write an ***informal* proof** for the **transitivity of identity**. Here was how I wrote out the informal proof in the answer key:

> We are told that $b = c$ and also that $a = b$. Using the first identity statement ($b = c$), by the indiscernibility of identicals we may substitute c for b in the second identity statement ($a = b$), giving us $a = c$ and proving the transitivity of identity.

→ Open **Exercise 2.16** in **Fitch**: Now you are asked to use Fitch to construct a ***formal*** proof for the same argument. **Open Fitch and construct the proof with the students**, explaining the features of Fitch along the way.

ANSWER:

1. b = c ← This premise can serve as 'instructions'
2. a = b ← cf. your goal. This premise can serve as 'template'
3. a = c = **Elim 1,2**

→ Open **Exercise 2.18** in **Fitch** and do with students.

ANSWER:

1. Between(a,d,b) ← notice similarity to goal. Only need to substitute 1st

2. a = c
3. e = b

and 3rd arguments of the predicate. Premise 2 allows switch from a to c, but Premise 3 is in the reverse order from what you need.

4. **e = e** **=Intro** ← use this step to create 'template'.
5. **b = e** **=Elim 4,3** ← this step gives you reverse of step 3.
6. **Between(c,d,b)** **=Elim 1,2** ← this makes the first substitution.
7. Between(c,d,e) **=Elim 6,5** ← this makes second substitution and gives you the goal.

→ Open **Exercise 2.20** in **Fitch**, which requires ***Ana Con***. Use this opportunity to **explain what analytical consequence** is (logical consequence based on the *meanings of the predicates* themselves).

ANSWER:

1. RightOf(b,c)
2. LeftOf(d,e)
3. b = d

The premises describe a spatial arrangement that looks like this:

c	b/ d	e

4. **LeftOf(c,b)** **Ana Con 1** ← reformats 1

compatible with goal.

5. **LeftOf(c,d)** **=Elim 4,3** ← makes the 'b to d' switch.
6. LeftOf(c,e) **Ana Con 2,5** ← transitivity follows from nature of the predicate involved.

PowerPoint slide #3

Pages 63-65 reiterates a point that I mentioned during last class, that to prove an argument ***invalid*** (i.e., proving ***non****consequence*) you find a ***counterexample*** showing that it is possible for the premises to be *true* but the conclusion *false*. The book gives the example of Al Gore:

> Al Gore is a politician.
> Hardly any politicians are honest.
> Therefore, Al Gore is dishonest.

Why is this argument not valid? [We can imagine a scenario in which Al Gore is the only honest politician out of 10,000]

PowerPoint slide #4

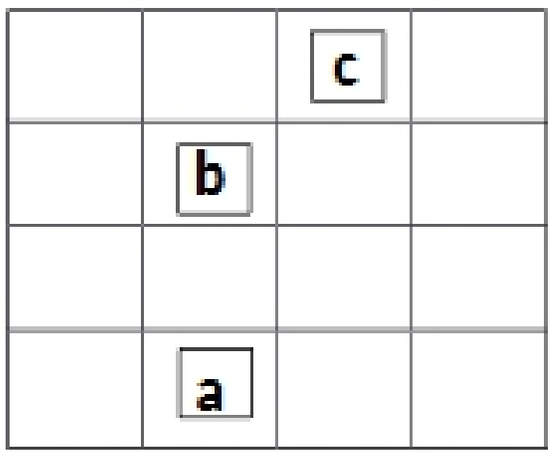
In this course, we will normally create **counterexample worlds** in Tarski's World, by building a block world that makes the premises of the argument true but the conclusion false.

Notice that the last section of your homework exercises for today (2.24-2.27) require you to ***either*** construct a proof of logical consequence in Fitch ***or*** build a counterexample world (as proof of nonconsequence) in Tarski's World for each exercise. (The ones that are valid require you to make liberal use of **ana con**.)

Powerpoint #5

Consider the argument in **exercise 2.25** together and decide whether it is valid or invalid.

Because it's invalid, we are required to build a counterexample world from scratch in **Tarski's World** with a configuration of objects similar to these below. (Open **Tarski's World** and build the world together)



← The fact that 'c' is to the right of 'a' but *in back of* 'b' allows the premises to be true but the conclusion *false*, making this a counterexample world.