<u>Answer Key</u> to select practice problems for lesson #16

I've included some explanations of basic points in the boxes beside each proof, but I don't discuss every single step in the proof, so you'll need to carefully study the proof itself to understand exactly what all is involved in it. If you have questions about specific steps of the proof (e.g., how a rule justifies that step, etc), feel free to email me with your (specific) questions or raise the question in class (if time permits).

<u>Proof 6.19</u>

1.	A v B	
2.	¬B v C	
3.	A	
4.	A v C	**v Intro:** 3
5.	B	
6.	¬B	
7.	⊥	**⊥ Intro:** 5,6
8.	A v C	**⊥ Elim:** 7
9.	C	
10.	A v C	**v Intro:** 9
11.	A v C	**v Elim:** 2, 6-8, 9-10
12.	A v C	**v Elim:** 1, 3-4,5-11

Notice that both premises and the conclusion are all disjunctions. Not surprisingly, **v Elim** turns out to be your overall strategy for this problem, one v **Elim** *inside* another **v Elim.** The first (and more basic) **v Elim** is based off premise#1, with A becoming the premise of the first subproof (at step 3) and B the premise of the second subproof (at step 5). You will reach your ultimate goal (i.e., A v C) as the common conclusion of these two subproofs (at steps 4 and 11), which will entitle you to assert that common conclusion at step 12 (remember, this is how **v Elim** works).

The trickiest part of this proof is that you need to construct a *second* **v Elim** strategy nested *inside* the second subproof mentioned above, which runs from steps 5-11. Notice that steps 6 and 9 each open a new sub-subproof with a premise that is taken from either of the two disjuncts in step 2. For this second **v Elim** strategy, you again prove the common conclusion A v C (at steps 8 and 10), which allows you to assert it one level back also at step 11 (via **v Elim**). And, as already mentioned above, step 11 turns out to be a common conclusion with step 4, which is part of your first, more basic v Elim strategy described in the first box above.

Proof 6.30

I once constructed a 23-step solution to this proof before someone pointed out to me that it could be done in 10 simple steps (well, maybe not so simple!). Here's the 10-step solution (I'd be embarrassed to show you my 23-step version—what a waste of ink!):

```
1.    ¬(¬Cube(a) ∧ Cube(b))
2.  | ¬(¬Cube(b) ∨ Cube(c))
    |
3.  |---- ¬Cube(a)
    |    |
4.  |    |---- ¬Cube(b)
    |    |    |
5.  |    |    |---- ¬Cube(b) ∨ Cube(c)      ∨ Intro: 4
6.  |    |    |     ⊥                       ⊥ Intro: 2,5
    |    |    |
7.  |    |     Cube(b)                      ¬ Intro: 4-6
8.  |    |     ¬Cube(a) ∧ Cube(b)           ∧ Intro: 3,7
9.  |    |     ⊥                            ⊥ Intro: 1,8
    |    |
10. |  Cube(a)                              ¬ Intro: 3-9
```

When you compare the two premises of this argument with the conclusion, there's no obvious way to 'extract' the conclusion (i.e., **Cube(a)**) from either of the premises. In such a case, the 'proof by contradiction' (**¬Elim**) strategy is often helpful to reach your goal, as it is here. To pursue a negation elim strategy, open a subproof with **¬Cube(a)** as its premise (at step 3), this being the *opposite* of what you want to actually conclude at step 10, namely, **Cube(a)**.

The challenge then becomes how to generate a contradiction (which you'll need for your **¬Elim** strategy) based on the premises you've got to work with. Notice that what you just premised at step 3 is actually half of an embedded conjunction in step 1 (see the **¬Cube(a)** conjoined in step 1 with **Cube(b)**). If you can build that conjunction **¬Cube(a) ∧ Cube(b))** *in your subproof*, you could then flag it as a contradiction to step 1 and complete your negation elim strategy.

Well, it turns out that we have a rule for building conjunctions (**∧ Intro**), so in addition to the **¬Cube(a)** you've premised at step 3, you just need to somehow generate the other half of the conjunction, **Cube(b).** That's where the nested subproof at steps 4-6 comes in: It's another **¬ elim** strategy *within* the larger **¬ elim** strategy. By premising at step 4 the opposite of what you need (i.e., premise **¬Cube(b)**), you can reach a contradiction at step 6 and assert **Cube(b)** one level back at step 7. Then, build the conjunction you need at step 8, flag the contradiction between steps 1 & 8 at step 9, and complete your overall **¬ elim** strategy with step 10.