

Summer MA 15200 Lesson 21 Section 4.2 and Applied Problems

Remember the following information about inverse functions.

1. In order for a function to have an inverse, it must be one-to-one and pass a horizontal line test.
2. The inverse function can be found by interchanging x and y in the function's equation and solving for y .
3. If $f(a) = b$, then $f^{-1}(b) = a$. The domain of f is the range of f^{-1} and the range of f is the domain of f^{-1} .
4. The compositions $f(f^{-1}(x))$ and $f^{-1}(f(x))$ both equal x .
5. The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

Because an exponential function is 1-1 and passes the horizontal line test, it has an inverse. This inverse is called a logarithmic function.

I Logarithmic Functions

According to point 2 above, we interchange the x and y and solve for y to find the equation of an inverse function.

$$f(x) = b^x \text{ exponential function}$$

$$x = b^y \text{ inverse function} \quad \text{How do we solve for } y? \text{ There is no way to do this.}$$

Therefore a new notation needs to be used to represent an inverse of an exponential function, the logarithmic function.

Definition of Logarithmic Function

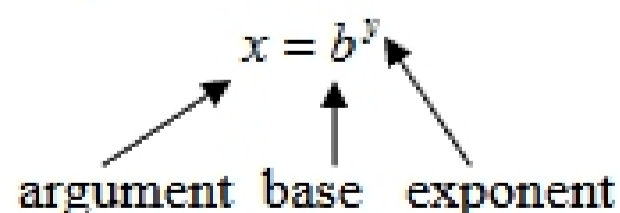
For $x > 0$ and $b > 0$ ($b \neq 1$)

$$y = \log_b x \text{ is equivalent to } x = b^y$$

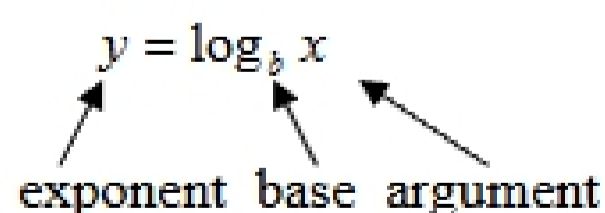
The function $f(x) = \log_b x$ is the **logarithmic function with base b** .

The equation $y = \log_b x$ is called the logarithmic form and the equation $x = b^y$ is called the exponential form. The value of y in either form is called a **logarithm**. Note: The logarithm is an exponent.

Exponential Form



Logarithmic Form



In this form, the y value representing the exponent is called a logarithm.

Ex 1: Convert each exponential form to logarithmic form and each logarithmic form to exponential form.

a) $3^4 = 81$

h) $m^p = x + 4$

b) $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

i) $(2a)^{y+7} = p^2$

c) $25^{\frac{1}{2}} = 5$

d) $8^{-2} = \frac{1}{64}$

e) $\log_2 32 = 5$

f) $\log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3$

j) $\log_q(2mn) = 12$

g) $\log_5 \sqrt{5} = \frac{1}{2}$

k) $\log_{x+3} 200 = rs$

II Finding logarithms

Remember: A logarithm is an exponent.

Ex 2: Find each logarithm.

a) $\log_{10} 100,000$

b) $\log_3 27$

c) $\log_{20} 1$

d) $\log_{15} 15$

e) $\log_{12} \frac{1}{144}$

$$f) \log_4 64$$

$$g) \log_{\frac{1}{2}} 32$$

$$h) \log_3 81$$

III Basic Logarithmic Properties

1. $\log_b b = 1$ Since the first power of any base equals that base, this is reasonable.
2. $\log_b 1 = 0$ Since any base to the zero power is 1, this is reasonable.

The exponential function $f(x) = b^x$ or $y = b^x$ and the logarithmic function $f^{-1}(x) = \log_b x$ or $y = \log_b x$ are inverses.

$$f(f^{-1}(x)) = x \text{ and } f^{-1}(f(x)) = x$$

This leads to 2 more basic logarithmic properties.

3. $\log_b b^x = x$ This is a composition function where $f(x) = b^x$ and $f^{-1}(x) = \log_b x$. $f^{-1}(f(x)) = f^{-1}(b^x) = \log_b b^x = x$ (the exponent)
4. $b^{\log_b x} = x$ This is a composition function where $f(x) = b^x$ and $f^{-1}(x) = \log_b x$. $f(f^{-1}(x)) = f(\log_b x) = b^{\log_b x} = x$ (the number or argument)

Ex 3: Simplify using the basic properties of logarithms.

$$a) \log_4 1 =$$

$$b) \log_3 3 =$$

$$c) 12^{\log_{12} 4} =$$

$$d) \log_{10} 10^5 =$$