LESSON 10: LOGICALTRUTH & TAUTOLOGIES

Assigned reading pp. 93-103

POWERPOINT SLIDE #1

LOGICAL POSSIBILITY:

A sentence-claim is ***logically** **possible*** if there is *some* (i.e., at least one) logically conceivable circumstance (situation or world) in which the claim is true.

> For example, ***"Jimmy Fallon is the president of the United States"***
> Though it's not true in *our* world, the 'real' world, we can at least *imagine* a 'possible' world in which it *might* be true; that is, there is nothing logically contradictory about the notion of Jimmy Fallon being president, say, in some parallel universe more or less like ours except with Fallon in the Oval Office.

POWERPOINT SLIDE #2

Proving logical possibility is simple: If you can conceive of a world (situation or state of affairs) in which the sentence is true, then you've demonstrated that it is logically possible. (Of course, the world has to abide by the laws of logic for this to count. So, for example, imagining a world in which multiple names of a *single* object are distributed onto *other* objects in the world would violate the indiscernibility of identicals and not be a 'logical' world for our purposes.)

POWERPOINT SLIDE #3

Logical possibility can also be *interpreted more narrowly* as being **specific to a particular world or set of worlds**, such as the worlds that can be built with the *Tarski's World* program (i.e., each world a particular block-inhabited checkered board).

> For example: ***Cube(a) ∧ Larger(a,b)***

Tarski's World has it rules and limits, so we won't find many imaginable situations realized in Tarski's World (e.g., you'll never see a Tarski's World world with ***tetrahedrons riding dancing ponies***).

Sentences that can be made true by *at least one* conceivable orientation of blocks in Tarski's World are called in the book ***TW-possible***.

POWERPOINT SLIDE #4

> To *prove* that a sentence is TW-possible, all you have to do is construct a world *in Tarski's World* that makes that sentence **true**.

For example, to show that *Cube(a) ∧ Larger(a,b)* is TW-possible, you can simply construct a world with a cube in it labeled *a* that is bigger than some other object labeled *b* in the same world).

POWERPOINT SLIDE #5

LOGICAL NECESSITY:

In the most general sense, a sentence-claim is ***logically necessary*** if it is true in ***every*** logically possible circumstance (in every logically conceivable world, at least in worlds that can still be considered 'logical'). Such a sentence is called a ***logical truth***.

For example, ***"Jimmy Fallon is Jimmy Fallon"*** is a logical truth, as long as we assume that the name 'Jimmy Fallon' in each case refers to the same individual (and we can assume this since equivocation of terms isn't allowed in FOL: that is, each individual constant can refer only and always to *one* individual, so as to rule out any ambiguity).

POWERPOINT SLIDE #6

Like logical possibility, logical necessity can be *interpreted more narrowly* as being **necessity relative to a particular world or set of worlds**, like those constructible in Tarski's World. Sentences that are true in every conceivable orientation of blocks in Tarski's World are called ***TW-necessary***. This may include some sentences that are necessary in the world merely because of the particular features or limits of that world.

For example, the sentence *Tet(a) v Cube(a) v Dodec(a)* is necessarily true in any Tarski's World world simply because *Tetrahedron*, *Cube*, and *Dodecahedron* are the ***only three shapes of objects available*** in the program.

Obviously, in most non-TW worlds (like our 'real' world), many other shapes of objects are possible, so a parallel sentence like "*Object* **a** *is a tetrahedron or a cube or a dodecahedron*" would ***not*** be logically necessary in those non-TW worlds (e.g., object 'a' could refer to my pet goldfish, which isn't a tet or cube or dodec—but my pet goldfish can't exist within any of the Tarski's World worlds).

There are ***two other kinds*** *of necessarily-true sentences in Tarski's World* that do ***not*** depend for their necessity on the unique properties of the TW program. Instead, these sentences are necessary for *strictly logical reasons* that hold both *inside* and *outside* of Tarski's World.

POWERPOINT SLIDE #7

At this point before going further we should look at **Figure 4.1** on **p. 102** of your textbook.

**** SHOW FIGURE 4.1 FROM TEXTBOOK ON THE SCREEN

Figure 4.1 diagrams the relationship between these different kinds of necessity (and possibility). **One important thing to remember about Figure 4.1** (so as not to get confused) is that it *describes only the sentences of Tarski's World*.

(as work through the diagram, refer to the sample sentences of FOL at each concentric level)...

(TW-) LOGICAL POSSIBILITIES

So, working from the outside in, outside the ***broadest* circle** in Figure 4.1 there is an unlabeled area that includes *every sentence that can* ***possibly be truly stated*** *of one or more 'worlds'* using the Tarski's World program. This is the area we have already talked about as being what is ***TW-possible***.

TW NECESSITIES

Just inside* this circle is a subset circle** that contains all the sentences stateable with Tarski's World that are ***necessarily true and labeled **"Tarski's World Necessities"** (= **TW-necessary**, as we've already discussed). These sentences will *always* be true in TW no matter how you move the blocks around, resize them, or reshape them. **Within the outermost part of this circle** are sentences like *Tet(a) v Cube(a) v Dodec(a)* that we discussed a moment ago—sentences that are necessarily true in TW because of the *peculiarities of the structure of TW* (i.e., in this case because there are only three shapes allowed in TW).

LOGICAL NECESSITIES (IN TW)

Within this circle is a smaller subset of sentences that Figure 4.1 simply calls **"Logical Necessities"**, meaning that the sentences within this circle are **logically necessary** or ***necessarily true for logical reasons*** that hold not just in Tarski's World but in all other logical worlds as well (including the 'real' world and any other logically conceivable possible world).

Keep in mind that even though logical necessity is a concept that holds *outside* of Tarski's World, Figure 4.1 is nonetheless *only describing the sentences of Tarski's World*, so the diagram can represent ***logically*** *necessary* sentences as being a subset of (i.e., a smaller group within) ***TW-****necessary* sentences. ***To avoid confusion,*** the circle labeled "Logical Necessities" in Figure 4.1 might be *better* labeled **"*Logical* Necessities *in TW*"**, as a reminder that there are many other logically necessary sentences (like "*Jimmy Fallon is Jimmy Fallon*") that are ***not*** part of Tarski's World and which therefore are ***not*** a subset of TW-necessary sentences.

There are different reasons for why a sentence (inside or outside of Tarski's World) might be logically necessary—that is, *necessarily true* on strictly ***logical*** grounds.

A sentence like *¬(Larger(a,b) ∧ Larger(b,a))* clearly expresses a logically necessary truth (i.e., it can never be false, on pain of contradiction, since all it states is the necessary fact that the two