LESSON 20: INTRODUCTION TO QUANTIFIERS (sections 9.1-9.4)

POWERPOINT SLIDE #1

Assigned reading pp. 229-241:

Examples of *quantification* (phrases that refer to quantities of things):

every cube
some students from USC
most vipers in the playpen
the dodecahedron in the bathroom
three blind mice
no student of logic
whenever

POWERPOINT SLIDE #2

Quantification is **not truth functional**.

For example, the truth or falsity of the sentence **"Every rich actor is a good actor"** cannot be determined by checking the truth values of parts of the sentence, not in the same way as you can check **"*b* is cube and *c* is a cube"**. Instead, the truth of the first sentence must be determined by the relationship between the collection of rich actors and the collection of good actors (i.e., by whether all members of the former set are also members of the latter set).

This is a new animal compared to what we've dealt with before.

POWERPOINT SLIDE #3

Two kinds of **terms**: ***individual constants*** (e.g., *a*, *b*, *claire*) and ***variables*** (e.g., *t*, *x*, y_{18},). Variables do *not* refer to specific objects the way individual constants do; variables are *placeholders* that indicate relationships between quantifiers and the arguments of predicates. (This last statement will make more sense soon . . .)

POWERPOINT SLIDE #4

The Universal Quantifier: ∀

means "every" (or "all")

∀x is translated "for every object *x* ..."

POWERPOINT SLIDE #5

∀x Home(x) = "For every object x, x is at home", or simply "everything is at home"

Binding: In the above expression we say that the universal quantifier ∀ ***binds*** the variable *x*. Without the quantifier present, as in just the expression Home(x), the variable is said to be ***unbound*** or ***free***.

POWERPOINT SLIDE #6

∀x (Doctor(x) → Smart(x)) = "Every doctor is smart" (literally: "For every x, if x is a doctor, then x is smart")

Scope: In the above sentence, the ***scope*** of the quantifier is shown by the outer parentheses; that is, the quantifier binds the x-variables on both the predicate 'Doctor' and the predicate 'Smart'.

POWERPOINT SLIDE #7

In the earlier example, ∀x Home(x), no parentheses are needed because the scope of the quantifier is simply over the one predicate Home and its variable *x*.

POWERPOINT SLIDE #8

The Existential Quantifier: ∃

Means "some" (i.e., at least one)

∃x = "for some object *x* …"

POWERPOINT SLIDE #9

∃x Home(x) = "For some object *x*, *x* is at home" or just "something is at home"

∃x (Doctor(x) ∧ Smart(x)) = "Some doctor is smart" or "There is at least one smart doctor"

POWERPOINT SLIDE #10

Binding and **scope** work exactly the same with the existential quantifier as they did with the universal quantifier.

POWERPOINT SLIDE #11

Be careful, though: The expression ∃x Doctor(x) ∧ Smart(x) looks misleadingly similar to the above example, but notice that it is missing the enclosing parentheses. This indicates that the scope of the existential quantifier in the latter example is only over Doctor(x) and NOT over Smart(x). We say that the first x-variable (as an argument of the predicate Doctor) is bound but the second x-variable (as an argument of Smart) is free.

POWERPOINT SLIDE #12

For an expression containing variables to be a **sentence**, *all of the variables must be bound by some or other quantifier.* If there are *any* unbound variables in the expression, the expression can at most be a **well-formed formula** (**wff**), *not* a sentence.

> **Well-formed formulas** (**wffs**): Expressions that have the proper formal *structure* of sentences of FOL (i.e., they contain predicates with terms as arguments—either constants or variables—and possibly quantifiers and connectives as well), all generated according to the grammatical rules of FOL, but possibly containing *unbound variables.*
>
> **Sentences:** Those wffs in which all variables (if any) are *bound*.

POWERPOINT SLIDE #13

On **pp. 233-234**, the texbook provides a list of the generative rules of FOL. Basically, these rules say that you can take *atomic wffs*, or wffs that have the same sort of structure as atomic sentences—as the wff **Doctor(x)** parallels the structure of **Doctor(max)**—and you can **negate** them or **combine** them with any of the **connectives** we've already studied (conjunction, disjunction, material conditional, biconditional) or place a **quantifier** in front of them. And you can do this over and over to construct increasingly complex wffs.

POWERPOINT SLIDE #14 (wffs that are *not* sentences)

So, taking everything we've said above into account, *all* of the following expressions #1-8 are **wffs** of FOL, but *only #5-8* are also **sentences**:

1. Cube(x) ∧ ¬Cube(y)
2. ∀x Doctor(x) → Smart(x)
3. ∃y RightOf (x,y)
4. (Cube(x) ∧ Small(x)) → ∃y LeftOf (x,y)

POWERPOINT SLIDE #15 (wffs that are also *sentences*)

5. Cube(a) ∧ Cube(b) — "*a* and *b* are cubes" [notice *a* and *b* are individual constants, not variables, so they don't need to be bound]
6. ∀x (Doctor(x) → Smart(x)) — "Every doctor is smart"
7. ∀x ((Cube(x) ∧ Small(x)) → ∃y LeftOf (x,y)) — "Every small cube is to the left of some object"
8. ∃z (Tet(z) ∧ Large(z)) — "There is a large tetrahedron"