LESSON #24: Review exercises for valid arguments (sec. 8.4)

Work together exercises from today's homework **on whiteboard**, in reverse order: **8.45, 8.47, 8.49, 8.51, 8.53**

Notice that for these exercises you must first decide whether the argument provided is valid or not. If it's valid, you should prove it formally. If it's invalid, you should provide a counterexample world.

Exercise 8.53

Small(a) → Small(b)
Small(b) → (SameSize(b,c) → Small(c))
¬Small(a) → (Large(a) ∧ Large(c))

SameSize(b,c) → (Large(c) ∨ Small(c))

How to figure out if this is valid or not?

If you can't wrap your head around the argument trying to see it as *valid*, try considering whether it could possibly be *invalid*—that is, would it be possible to construct a counterexample? A counterexample would make the premises all *true* but the conclusion *false*.

Now, focus on the **conclusion** for a moment: it is a conditional statement, which means that the only way it can be false is if its *antecedent* is *true* but its *consequent* is *false* (recall the truth table for the material conditional). That would mean that in the counterexample world we're trying to construct, *b* and *c* would be the *same size*, but both would have to be *medium* (because to make the consequent false, *c* could be neither large nor small, which leaves only medium, which then means that both it and *b* would have to be medium). So, in the counterexample world (assuming we can succeed in constructing it), *b* and *c* would both be medium.

In this same counterexample world, all three *premises* of the argument must be *true*. Let's see if this is possible to do for a world in which *b* and *c* are both medium (so that the conclusion of the argument can be false). The premises all together deal with two basic possibilities: one where *a* is small and the other option where *a* is not small. The first case is covered by the first two premises, though the only one we really need to consider is premise #1: If *a* is small, *b* is also small. But this can't be the case for the potential counterexample world we're trying to construct, because we already know from considering the conclusion of the argument that in this counterexample world *b* and *c* must both be *medium* (see the paragraph above). So, if our counterexample world is going to make the conclusion of the argument false but all the premises true (as it must, in order to function as a counterexample), *a cannot* be small (that way, we can avoid the first premise ever being triggered to where it would require that *b* be small).

We can move on then, to the third premise, which covers the case where *a* is *not* small, and it tells us that in that case, *a* and *c* would both be *large*. Uh-oh, we've slammed into a wall. We saw earlier that the only way to make the conclusion of the argument false (as it must be in a genuine counterexample world) is for *c* to be *medium*, not large (for reasons we've already discussed above). So, our only remaining way of making the premises true (i.e., by assuming that in the counterexample world *a* is *not* small) has failed (because if *a* isn't small, premise #3 says it must be large, but this contradicts what we need in order to make the conclusion of the argument false for the purpose of creating a counterexample world). What we've discovered through all this is that there is no way to make all the premises of this argument *true* in some world while making the conclusion *false* in the same world. In other words, it is impossible to build a counterexample world to prove this argument invalid. And *that* means the argument is *valid!*

Now prove it.

The conclusion of the argument is a conditional, which tells us that we need to open a subproof beginning at step 4 premising **SameSize(b,c)** and concluding one step before the ultimate goal with **Large(c) v Small(c).** The question then becomes how to traverse the distance between these two steps.

We received one clue while we were looking at the problem and trying to build a counterexample: Recall that in light of the premises we had to consider *two cases*, the first case where *a* is *small* and the second case where *a* is *not* small. The premises, then, suggest a 'proof by cases' strategy; however, the problem is that we have *no disjunction* to work with in the premises (and we must have a disjunction to cite if we're going to use **v Elim**).

But there is a ray of hope: the disjunction we have in mind, basically **Small(a) v ¬Small(a)**, is a *tautology* (it's an instance of the law of excluded middle), so we should be able to generate it in our proof *without having to rely on any premises*. (Actually, there is an easier way to do this using the **Taut Con** 'shortcut' rule, but I'd rather us do it the hard way.) The trick is to use several applications of the **¬ Intro** strategy . . . I'll show you how in class (though you might be able to figure it out on your own).

Then, once we have the disjunction above as part of our proof, we can set up a proof by cases and use the information in the premises to where *a* is small or not small, either way leads us to the common conclusion **Large(c) v Small(c).** At that point we'll have proven the conditional relation reflected in the ultimate goal and can use the **→ Intro** rule to justify this conclusion. I'll show you all this in class step-by-step, but this is the overall outline of our strategy.

POWERPOINT SLIDE 1 (provides complete solution)

<u>Exercise 8.51</u>

| Cube(b) ↔ (Cube(a) ↔ Cube(c))

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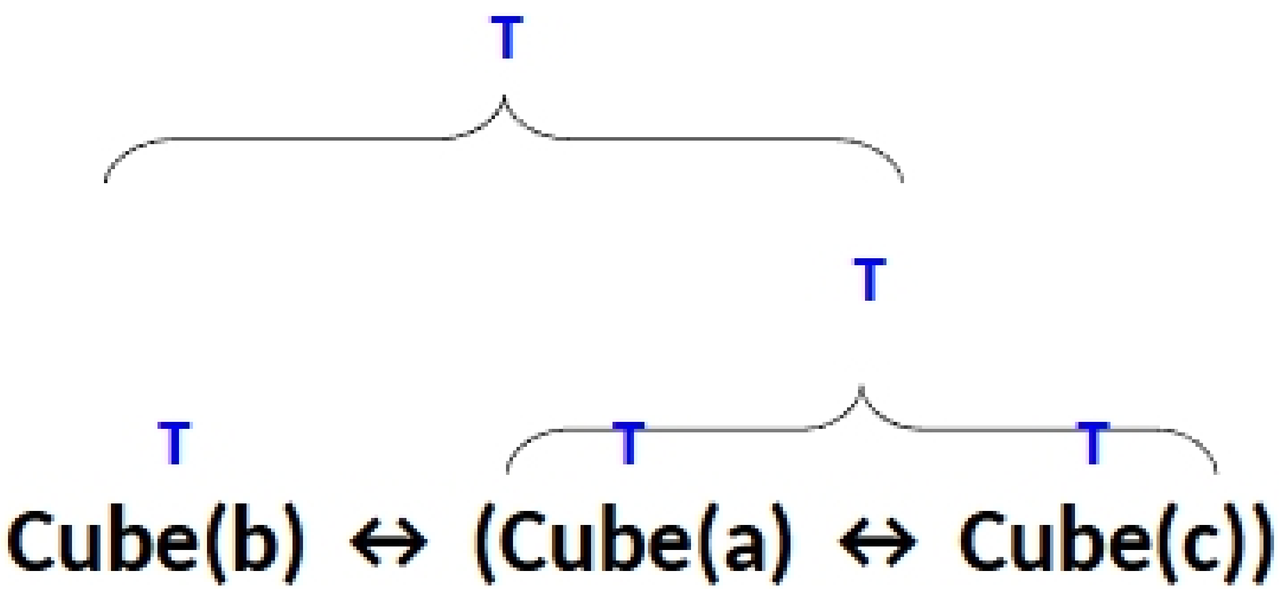
| Dodec(b) → a ≠ b

What would it mean for the premise here to be true? Keep in mind that it is a complex biconditional, and each biconditional entails a two-way conditional relationship. So, the two sides of the main biconditional (i.e., **Cube(b)**, on the one hand, and **(Cube(a) ↔ Cube(c))** on the other) must *covary* in their truth values for the main biconditional—i.e., the entire sentence—to be true. One obvious way this could be the case is if *a*, *b*, and *c*, are *all cubes* in a world. That situation can be schematized as follows, where the truth values of the atomic parts would combine—in this case very straightforwardly—to yield the truth value of the entire sentence (in a manner similar to a truth table):



Another way that the entire sentence could be true is for the second half of the main biconditional, namely, the phrase **(Cube(a) ↔ Cube(c))**, to be true by virtue of **neither** *a* **nor** *c* being a cube (i.e., a F + F combination in a biconditional yields a T for the whole phrase, because F + F covaries—what matters here is that they're the *same* F value). That situation would schematize as follows: