LESSON #21: MATERIAL CONDITIONAL (7.1); BICONDITIONAL (7.2); INFORMAL METHODS OF PROOF (8.1)

Assigned reading pp. 178-185

I'm going to skip some of the discussion that's in the text about the nature of truth-functional connectives, though it is still important and you should read and understand it. But I think our time here is best spent if I focus on the most important ideas in the text.

POWERPOINT SLIDE #1

The Material Conditional →

Translates as "If P, then Q", where P and Q represent two sentences of FOL, and where P is the 'antecedent' or the *condition* that—when met or satisfied—ensures that Q (the 'consequent') follows as as a logical consequence. You can also read this less formally as "Whenever sentence P is true (in some world), sentence Q will also be true (in that world)."

$$P \rightarrow Q$$

antecedent consequent

POWERPOINT SLIDE #2

Discuss the truth table for the material condition relation. Notice that the sentence is *false* in only **one** case (i.e., when the condition is met but the consequence doesn't follow), and that . . . and this may seem a little weird . . . **a material conditional with a *false antecedent* is always *true* as a whole sentence**.

POWERPOINT SLIDE #3

There are other English expressions that can be translated with the material conditional:

P → Q

"if P then Q"	"**if P**, then Q"
"if P, Q"	"**if P**, Q"
"Q if P"	"Q **if P**"
"Q provided P"	"Q **provided P**"

"provided P, Q"　　　　　　"**provided P**, Q"

Notice that in all these cases, **P** occurs in the English translation immediately *after* the word "if" (or its equivalent "provided")—whereas in the FOL sentence P comes *before* the conditional arrow. Obviously, then, you can't simply translate the material conditional arrow symbol solely with the word "if" without getting youself in trouble. It's safer to think of it as indicating the whole "if ... then ..." relation. Like this:

POWERPOINT SLIDE #4

P → Q		**P → Q**
if ... then ...	*NOT:*	... if ...

POWERPOINT SLIDE #5

"only if": There is one exception to what I've said above, namely, when you see "only if". Notice that here the order of P and Q relative to the word "if" is *reversed*, which means that "only if" is the only translation of the bunch where you can read the conditional arrow → straight off before the consequent:

"P **only if** Q"

P → Q

This is because, as the book explains on p. 182, adding the word "only" highlights the fact that there is a ***necessary*** conditional relationship holding between P and Q versus merely a *sufficient* conditional relationship.

I'm not going to go further into this in class because it can be confusing and the point is not important enough to spend time on. Just remember that "only if" is exceptional.

POWERPOINT SLIDE #6

The English word "unless" translates the into FOL as an "if not" relation:

"**unless P**, Q"

"Q **unless P**"

"**If not P**, Q"

"Q **if not P**"

All these are translations of: $\neg P \rightarrow Q$

POWERPOINT SLIDE #7

The **Biconditional**: **P ↔ Q**

This is when P and Q **always have the same truth values** (i.e., when one is T, the other is T, and when one is F, the other is F).

This is just like the regular material conditional above, except the relation works ***both ways***.

Usually translated as **"if and only if"** and abbreviated as **"iff"** (notice that there are *two* f's in the abbreviated form. **This is <u>not</u> a typo!**). Also can be phrased "just in case" (mathematician's usage).

POWERPOINT SLIDE #8

The truth table for the biconditional reflects the fact that the truth values of the two halves of a biconditional statement always covary. So, a biconditional is true only when P and Q are *both true* (row 1), or when they are *both false* (the last row).

POWERPOINT SLIDE #9

Note that the **biconditional** bears a close resemblance to the **logical equivalence relation** we've already learned, except that whereas equivalence is a *relation between multiple sentences*, the biconditional material is a *connective* used *within one sentence* at a time.

There is, however, a way to express the relationship between the biconditional and the equivalence relation: Two sentences P and Q are **logically equivalent** if and only if the **biconditional** formed from them, P ↔ Q is a **<u>logical truth</u>** (i.e., true in every world, or can never be false).