LESSON EIII.E – EXPONENTS AND LOGARITHMS

OVERVIEW

Here's what you'll learn in this lesson:

Exponential Functions

a. Graphing exponential functions

b. Applications of exponential functions

c. Solving some exponential equations

Logarithmic Functions

a. Exponential and logarithmic form

b. Graphing logarithmic functions

c. Properties of logarithms

Solving Equations

a. Using a calculator to approximate common and natural logarithms

b. Change of base formula

c. Solving logarithmic equations

d. Solving exponential equations

Exponential and logarithmic functions have many useful applications in fields ranging from investment banking to medicine. They are used to measure many things ranging from the strength of an earthquake to the noise level in a recording studio.

In this lesson, you will look at some applications as you review exponential and logarithmic functions. You will start by graphing them. You will also review the properties of exponents and logarithms, and you will see how these properties can be used to solve exponential equations.

EXPLAIN

EXPONENTIAL FUNCTIONS

Summary

You have already graphed linear functions and quadratic functions. In this concept you will work with exponential functions. You will graph exponential functions, look at applications of exponential functions, and solve exponential equations.

Definition of an Exponential Function

Here are some examples of exponential functions:

$f(x) = 5^x$ $\qquad g(x) = \left(\frac{1}{3}\right)^x$ $\qquad h(x) = 7^{-x}$

In general, an exponential function is a function of the form $y = f(x) = b^x$.

Here, the constant b is called the base and is a positive number not equal to 1. The independent variable, x, is the exponent.

The domain of an exponential function is all real numbers.

The range of an exponential function is all positive real numbers.

Notice that in an exponential function, the variable is the exponent. The function $y = x^3$ is not an exponential function because the variable, x, is the base.

The Graph of an Exponential Function

To graph an exponential function you can make a table of points, plot the points, and join them with a smooth curve, as you have done for other functions.

Here's a table of points for the exponential function $y = 3^x$. The graph is shown in Figure EIII.E.1.

x	$y = 3^x$
3	27
2	9
1	3
0	1
–1	$\frac{1}{3}$
–2	$\frac{1}{9}$
–3	$\frac{1}{27}$

From the graph of the exponential function $y = 3^x$, you can see that as x increases, the graph rises rapidly. As x becomes more negative, the graph gets closer to the x-axis, but never becomes zero or negative. The y-intercept is the point (0, 1).

The variable, x, does not have to be a whole number. For example, when $x = \frac{1}{2}$, $f(x) = 3^{\frac{1}{2}} = \sqrt{3}$. You can use your calculator to approximate $\sqrt{3} = 1.73$.

Remember, $3^{-2} = \frac{1}{3^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$.

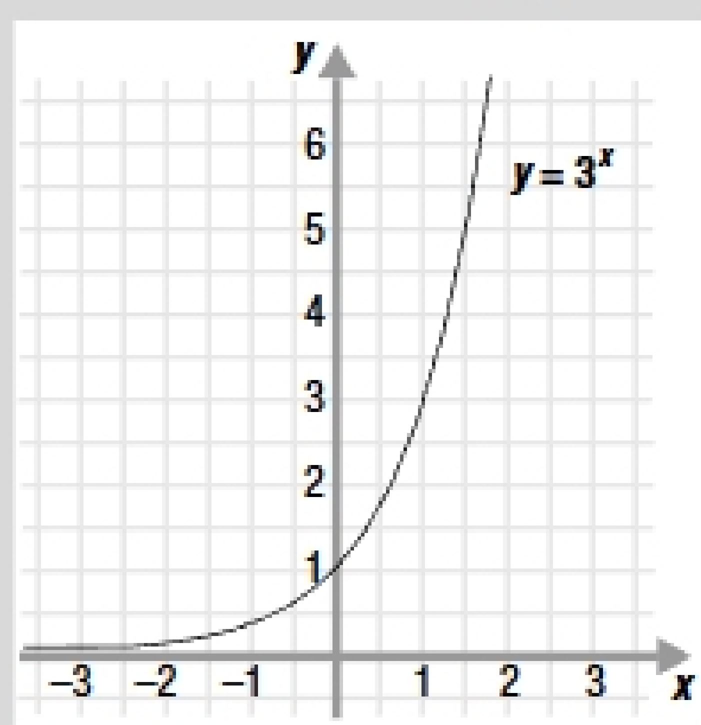

Figure EIII.E.1