## Section 1.1: Statements and Conditional Statements

**Statement**: a declarative sentence that is either true of false but not both.

-also called a proposition

-to establish truth→ write mathematical proof

-to establish false→ provide counterexample

**Techniques of Exploration**

-guesswork and conjectures

-Examples. Constructing appropriate examples is very important.

-can only say appears to be true if you can't find a counter

-Use of prior knowledge

-cooperation and brainstorming

**Conditional Statements:** given the form "If P, then Q", where P and Q are sentences. For this conditional statement, P is called the hypothesis and Q is called the conclusion.

| $P$ | $Q$ | $P \to Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

*Think of If P then Q to be false if I lied and to be true If I did not lie.

**Rational numbers (Q):** are those real numbers that can be written as a quotient of two integers.

**Irrational numbers:** are those real numbers that cannot be written as a quotient of two integers.

**Natural numbers (N):** consist of the positive whole numbers.

**Integers (Z):** consist of zero, the positive whole numbers, and the negatives of the positive whole numbers. *each integer is a rational and a real number*

-integers are closed under addition, multiplication, and subtraction.

## Section 1.2: Constructing Direct Proofs

**Definition:** an agreement that a particular word or phrase will stand for some object, property, or other concept that we expect to refer to often.

**Even Integer:** an integer a is even provided that there exists an integer n such that a=2n.

**Odd Integer:** an integer a is odd provided that there exists an integer n such that a=2n+1.

For all real numbers $x$, $y$, and $z$

| | |
|---|---|
| Identity Properties | $x + 0 = x$ and $x \cdot 1 = x$ |
| Inverse Properties | $x + (-x) = 0$ and if $x \neq 0$, then $x \cdot \frac{1}{x} = 1$. |
| Commutative Properties | $x + y = y + x$ and $xy = yx$ |
| Associative Properties | $(x + y) + z = x + (y + z)$ and $(xy)z = x(yz)$ |
| Distributive Properties | $x(y + z) = xy + xz$ and $(y + z)x = yx + zx$ |

*Know-Show Table: steps…

1. Identify the hypothesis, P (first step), and the conclusion, Q (goal), of the conditional statement.

2. Start with things we know, such as definitions

3. ask backward and forward questions

4. use algebra/substitution/whatever to finish proving

**Mathematical Proof:** a convincing argument that a certain mathematical statement is necessarily true.

**Guidelines for Mathematical proof**

1. Begin with a carefully worded statement of the theorem or result to be proven.
   a. State theorem
   b. Skip a line and write "Proof" in italics or bold
2. Begin the proof with a statement of your assumptions
   a. "we assume that…"
3. Use the pronoun "we"
4. Use italics for variables when using word processor
5. Display important equations and mathematical expressions
   a. Algebra or mathematical expressions should be separated by a space and centered
6. Tell the reader when the proof has been completed
   a. "this completes the proof" or end of proof symbol ▪

Section 2.1: Statements and Logical Operators

**Logical operator:** on mathematical statements, is a word or combination of words that combines one or more mathematical statements to make a new mathematical statement.

**Compound statement:** a statement that contains one or more operators

-Conjunction→ "P and Q" or P^Q. Only true when both P and Q are true

-Disjunction→ "P or Q" or PvQ. Only true when at least one of Por Q is true. False only when both are false

-Negation→ "not P" or ~P. negation of P is true only when P is false and vice versa.

-implication→ "If P then Q" or P→Q. Only false when P is true and Q is false

**Other ways to expresses P→ Q...**

-If P, then Q -Q if P -P implies Q

-Whenever P is true, Q is true. -P only if Q -Q is true Whenever P is true.

-Q is necessary for P. (means that if P is true, then Q is necessarily true. "is equivalent to")

-P is sufficient for Q (means if you want Q to be true, it is sufficient to show P is true. A reason for a conclusion but not the only reason)

***know how to construct truth table***

**Biconditional statement:** "P if and only if Q" or P←→ Q or [(Q→P) ^ (P→Q)]

-can be expressed as... P if and only if Q, P is necessary and sufficient for Q, and P implies Q and Q implies P.

**Tautology:** a compound statement S that is true for all possible combinations of truth values of the component statements that are part of S.

**Contradiction:** a compound statement that is false for all possible combinations of truth values of truth values of the component statements that are part of S

Section 2.2: Logically Equivalent Statements

**Logically equivalent:** two statements are this provided that they have the same truth values for all possible combinations of truth values for all variables appearing in the two expressions. In this case we write X≡Y and say that X and Y are logically equivalent.

**Converse:** of conditional statement P→Q is the conditional statement Q→P

**Contrapositive:** of the conditional statement P→ Q is the conditional statement ~Q→~P