

CS 416 Artificial Intelligence

Lecture 13
First-Order Logic
Chapter 8

Question about Final Exam

- I will have a date for you by Tuesday of next week.

First-order logic

- We saw how propositional logic can create intelligent behavior
- But propositional logic is a poor representation for complex environments
 - Why?
- First-order logic is a more expressive and powerful representation

What do we like about propositional logic?

- It is:
 - Declarative
 - Relationships between variables are described
 - A method for propagating relationships
 - Expressive
 - Can represent partial information using disjunction
 - Compositional
 - If A means foo and B means bar, $A \wedge B$ means foo and bar

What don't we like about propositional logic?

- Lacks expressive power to describe the environment concisely
 - Separate rules for every square/square relationship in Wumpus world

Natural Language

- English appears to be expressive
 - Squares adjacent to pits are breezy
- But natural language is a medium of communication, not a knowledge representation
 - Much of the information and logic conveyed by language is dependent on context
 - Information exchange is not well defined
 - Not compositional (combining sentences may mean something different)
 - It is ambiguous

But we borrow representational ideas from natural language

- Natural language syntax
 - Nouns and noun phrases refer to objects
 - People, houses, cars
 - Verbs and verb phrases refer to relationships btw objects
 - Red, round, nearby, eaten
 - Some relationships are clearly defined functions where there is only one output for a given input
 - Best friend, first thing, plus
- We build first order logic around objects and relations

Ontology

- a "specification of a conceptualization"
- A description of the objects and relationships that can exist
 - Propositional logic had only true/false relationships
 - First-order logic has many more relationships
- The ontological commitment of languages is different
 - How much can you infer from what you know?
 - Temporal logic defines additional ontological commitments because of timing constraints

Higher-order logic

- First-order logic is "first" because you relate objects (the first-order entities that actually exist in the world)
 - There are 10 chickens... chickens.number=10
 - There are 10 ducks... ducks.number=10
- You cannot build relationships between relations or functions
 - There are as many chickens as ducks... chickens.number = ducks.number
 - the number of objects belonging to a group must be a property of the group, and not the objects themselves
 - Cannot represent Leibniz's law: If x and y share all properties, x is y

Another characterization of a logic

- Epistemological commitments
 - The possible states of knowledge permitted with respect to each fact
 - In first-order logic, each sentence is a statement that is
 - True, false, or unknown

Formal structure of first-order logic

- Models of first-order logic contain:
 - A set of objects (its domain)
 - Alice, Alice's left arm, Bob, Bob's hat
 - Relationships between objects
 - Represented as tuples
 - Sibling (Alice, Bob), Sibling (Bob, Alice)
 - On head (Bob, hat)
 - Person (Bob), Person (Alice)
 - Some relationships are functions if a given object is related to exactly one object in a certain way
 - Alice \rightarrow Alice's left arm

First-order logic syntax

- Constant Symbols
 - A, B, Bob, Alice, Hat
- Predicate Symbols
 - Is, onHead, hasColor, person
- Function Symbols
 - Mother, leftLeg
- Each predicate and function symbol has an arity
 - A constant the fixes the number of arguments

First-order logic syntax

- Names of things are arbitrary
 - Knowledge base adds meaning
- Number of possible domain elements is unbounded
 - Number of models is unbounded
 - Checking enumeration by entailment is impossible

Syntax

- Term
 - A logical expression that refers to an object
 - Constants
 - We could assign names to all objects, like providing a name for every shoe in your closet
 - Function symbols
 - Used in place of a constant symbol `OnLeftFoot(John)`

Atomic Sentences

- Formed by a predicate symbol followed by parenthesized list of terms
 - `Sibling(Alice, Bob)`
 - `Married(Father(Alice), Mother(Bob))`
- An atomic sentence is true in a given model, under a given interpretation, if the relation referred to by the predicate symbol holds among the objects referred to by the arguments

Complex sentences

- We can use logical connectives
 - `~Sibling(LeftLeg(Alice), Bob)`
 - `Sibling(Alice, Bob) ^ Sibling(Bob, Alice)`

Quantifiers

- A way to express properties of entire collections of objects
 - Universal quantification (\forall all,)
 - The power of first-order logic
 - $\forall x, \text{King}(x) \Rightarrow \text{Person}(x)$

Universal Quantification

- For all x , P
 - P is true for every object x
 - For all x , $\text{King}(x) \Rightarrow \text{Person}(x)$
 - Richard the Lionheart
 - King John
 - Richard's left leg
 - John's left leg
 - The crown

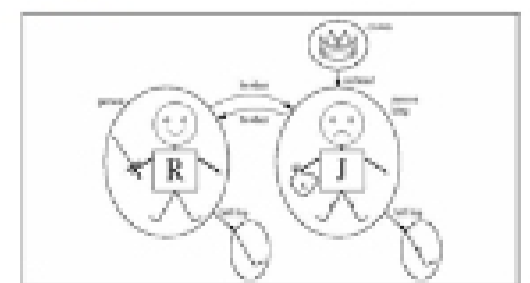


Figure 4.1 A model containing the objects, the binary relations, their many relations (left and right) between the objects, and the many binary relations.