

## CS 416 Artificial Intelligence

Lecture 15  
First-Order Logic  
Chapter 9

## Guest Speaker

- Topics in Optimal Control, Minimax Control, and Game Theory
  - March 28<sup>th</sup>, 2 p.m. OLS 005
  - Onesimo Hernandez-Lerma
  - Department of Mathematics
  - CINVESTAV-IPN, Mexico City
- This is a nontechnical introduction, mainly thru examples, to some recent topics in control and game theory, including adaptive control, minimax control (a.k.a. "worst-case control" or "games against nature"), partially observable systems (a.k.a. controlled "hidden Markov models"), cooperative and noncooperative game equilibria, etc.

## Final Exam

- Final Exam will be May 6<sup>th</sup> at 7:00 p.m.
- This conflicts with the fewest number of other exams

## Forward Chaining

- Remember this from propositional logic?
  - Start with atomic sentences in KB
  - Apply Modus Ponens
    - add new sentences to KB
    - discontinue when no new sentences
  - Hopefully find the sentence you are looking for in the generated sentences

## Lifting forward chaining

- First-order definite clauses
  - all sentences are defined this way to simplify processing
    - disjunction of literals with exactly one positive
    - clause is either atomic or an implication whose antecedent is a conjunction of positive literals and whose consequent is a single positive literal

$King(x) \wedge Greedy(x) \Rightarrow Evil(x)$   
 $King(John) .$   
 $Greedy(y) .$

## Example

- The law says it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American
- We will prove West is a criminal

## Example

- It is a crime for an American to sell weapons to hostile

**nations**  
 $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

- Nono... has some missiles

- Owns (Nono, M1)
- Missile (M1)

- All of its missiles were sold to it by Colonel West

$Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

## Example

- We also need to know that missiles are weapons

$Missile(x) \Rightarrow Weapon(x)$

- and we must know that an enemy of America counts as "hostile"

$Enemy(x, America) \Rightarrow Hostile(x)$

- "West, who is American"

$American(West)$

- The country Nono, an enemy of America

$Enemy(Nono, America)$

## Forward-chaining

- Starting from the facts

- find all rules with satisfied premises
- add their conclusions to known facts
- repeat until
  - query is answered
  - no new facts are added

## First iteration of forward chaining

- Look at the implication sentences first

$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

- must satisfy unknown premises

- We can satisfy this rule

$Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

- by substituting {x/M1}
- and adding Sells(West, M1, Nono) to KB

## First iteration of forward chaining

- We can satisfy

$Missile(x) \Rightarrow Weapon(x)$

- with {x/M1}
- and Weapon (M1) is added

- We can satisfy

•  $Enemy(x, America) \Rightarrow Hostile(x)$

- and Hostile (Nono) is added

## Second iteration of forward chaining

- We can satisfy

$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

- with {x/West, y/M1, z/Nono}
- and Criminal (West) is added

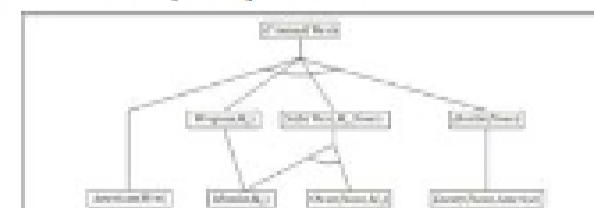


Figure 9.4 The proof tree generated by forward chaining on the crime example. The initial facts appear at the bottom level, facts inferred on the first iteration in the middle level, and facts inferred on the second iteration at the top level.

## Analyze this algorithm

- Sound?
  - Does it only derive sentences that are entailed?
  - Yes, because only Modus Ponens is used and it is sound
- Complete?
  - Does it answer every query whose answers are entailed by the KB?
  - Yes if the clauses are definite clauses

## Proving completeness

- Assume KB only has sentences with no function symbols
  - What's the most number of iterations through algorithm?
  - Depends on the number of facts that can be added
    - Let  $k$  be the arity, the max number of arguments of any predicate and
    - Let  $p$  be the number of predicates
    - Let  $n$  be the number of constant symbols
  - At most  $pn^k$  distinct ground facts
  - Fixed point is reached after this many iterations
  - A proof by contradiction shows that the final KB is complete

## Complexity of this algorithm

- Three sources of complexity
  - inner loop requires finding all unifiers such that premise of rule unifies with facts of database
    - this "pattern matching" is expensive
  - must check every rule on every iteration to check if its premises are satisfied
  - many facts are generated that are irrelevant to goal

## Pattern matching

- Conjunct ordering
  - Missile( $x$ )  $\wedge$  Owns(Nono,  $x$ )  $\Rightarrow$  Sells(West,  $x$ , Nono)
    - Look at all items owned by Nono, call them  $X$
    - for each element  $x$  in  $X$ , check if it is a missile
  - Look for all missiles, call them  $X$
  - for each element  $x$  in  $X$ , check if it is owned by Nono
- Optimal ordering is NP-hard, similar to matrix mult

## Incremental forward chaining

- Pointless (redundant) repetition
  - Some rules generate new information
    - this information may permit unification of existing rules
  - some rules generate preexisting information
    - we need not revisit the unification of the existing rules
- Every new fact inferred on iteration  $t$  must be derived from at least one new fact inferred on iteration  $t-1$

## Irrelevant facts

- Some facts are irrelevant and occupy computation of forward-chaining algorithm
  - What if Nono example included lots of facts about food preferences?
    - Not related to conclusions drawn about sale of weapons
    - How can we eliminate them?
      - Backward chaining is one way