

Introduction to Logic

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Notes

Introduction I

Propositional calculus (or logic) is the study of the logical relationship between objects called propositions and forms the basis of all mathematical reasoning and all automated reasoning.

Definition

A *proposition* is a statement that is either *true* or *false*, but not both (we usually denote a proposition by letters; p, q, r, s, \dots).

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Introduction II

Definition

The value of a proposition is called its *truth value*; denoted by T or 1 if it is true and F or 0 if it is false.

Opinions, interrogative and imperative sentences are not propositions.

Truth table:

P
0
1

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Examples I

Example (Propositions)

- ▶ Today is Monday.
- ▶ The derivative of $\sin x$ is $\cos x$.
- ▶ Every even number has at least two factors.

Example (Not Propositions)

- ▶ C++ is the best language.
- ▶ When is the pretest?
- ▶ Do your homework.

Notes

Examples II

Example (Propositions?)

- ▶ $2 + 2 = 5$
- ▶ Every integer is divisible by 12.
- ▶ Microsoft is an excellent company.

Notes

Logical Connectives

Connectives are used to create a *compound proposition* from two or more other propositions.

- ▶ Negation (denoted \neg or !)
- ▶ And (denoted \wedge) or Logical Conjunction
- ▶ Or (denoted \vee) or Logical Disjunction
- ▶ Exclusive Or (XOR, denoted \oplus)
- ▶ Implication (denoted \rightarrow)
- ▶ Biconditional; "if and only if" (denoted \leftrightarrow)

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Negation

A proposition can be negated. This is also a proposition. We usually denote the negation of a proposition p by $\neg p$.

Example (Negated Propositions)

- ▶ Today is not Monday.
- ▶ It is not the case that today is Monday.
- ▶ It is not the case that the derivative of $\sin x$ is $\cos x$.

Truth table:

p	$\neg p$
0	1
1	0

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Logical And

The logical connective AND is true only if both of the propositions are true. It is also referred to as a *conjunction*.

Example (Logical Connective: AND)

- ▶ It is raining and it is warm.
- ▶ $(2 + 3 = 5) \wedge (\sqrt{2} < 2)$
- ▶ Schrödinger's cat is dead and Schrödinger's cat is not dead.

Truth table:

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

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Logical Or

The logical disjunction (or logical or) is true if one or both of the propositions are true.

Example (Logical Connective: OR)

- ▶ It is raining or it is the second day of lecture.
- ▶ $(2 + 2 = 5) \vee (\sqrt{2} < 2)$
- ▶ You may have cake or ice cream.¹

Truth table:

p	q	$p \wedge q$	$p \vee q$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

¹Can I have both?

Notes
