

# Propositional Logic

## Chapter 7

### Outline

- Review
  - Knowledge-based agents
  - Logic in general
  - Propositional logic in particular – syntax and semantics
- Wumpus world
- Inference rules and theorem proving
  - Resolution
  - forward chaining
  - backward chaining

## Logic in general

- **Logics** are formal languages for representing information such that conclusions can be drawn
- **Syntax** defines the sentences in the language
- **Semantics** define the "meaning" of sentences;
  - i.e., define **truth** of a sentence in a world
- E.g., the language of arithmetic
  - $x+2 \geq y$  is a sentence;  $x^2+y > \{ \}$  is not a sentence
  - $x+2 \geq y$  is true iff the number  $x+2$  is no less than the number  $y$
  - $x+2 \geq y$  is true in a world where  $x = 7, y = 1$
  - $x+2 \geq y$  is false in a world where  $x = 0, y = 6$

## Entailment

- **Entailment** means that one thing **follows from** another:
$$KB \models \alpha$$
- Knowledge base  $KB$  entails sentence  $\alpha$  if and only if  $\alpha$  is true in all worlds where  $KB$  is true
  - E.g., the KB containing "the Steelers won" and "the Bengals won" entails "Either the Steelers won or the Bengals won"
  - E.g.,  $x+y = 4$  entails  $4 = x+y$
  - Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**

A	B	C	$A \wedge B$	$A \wedge C$	$B \wedge C$
F	F	F	F	F	F
F	F	T	F	F	F
F	T	F	F	F	F
F	T	T	F	F	T
T	F	F	F	F	F
T	F	T	F	T	F
T	T	F	T	F	F
T	T	T	T	T	T

$A \wedge C, C$   
does not  
entail  
 $B \wedge C$

$A, B,$   
Entails  
 $A \wedge B$

## Inference

- $KB \vdash_i \alpha$  = sentence  $\alpha$  can be derived from  $KB$  by procedure  $i$
- **Soundness:**  $i$  is sound if whenever  $KB \vdash_i \alpha$ , it is also true that  $KB \models \alpha$
- **Completeness:**  $i$  is complete if whenever  $KB \models \alpha$ , it is also true that  $KB \vdash_i \alpha$
- **Preview:** we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
- That is, the procedure will answer any question whose answer follows from what is known by the  $KB$ .