Matched, Lossless, Reciprocal Devices

As we discussed earlier, a device can be **lossless** or **reciprocal**. In addition, we can likewise classify it as being **matched**.

Let's examine **each** of these three characteristics, and how they relate to the **scattering matrix**.

Matched

A matched device is another way of saying that the **input impedance** at each port is **equal to** Z_0 when **all other** ports are terminated in matched loads. As a result, the **reflection coefficient** of each port is **zero**—no signal will be come out of a port if a signal is incident on that port (but **only** that port!).

In other words, we want:

$$V_m^- = S_{mm} V_m^+ = 0 \quad \text{for all } m$$

a result that occurs when:

$$S_{mm} = 0 \quad \text{for all } m \text{ if matched}$$

We find therefore that a matched device will exhibit a scattering matrix where all **diagonal elements** are **zero**.

Therefore:

$$\mathcal{S} = \begin{bmatrix} 0 & 0.1 & j0.2 \\ 0.1 & 0 & 0.3 \\ j0.2 & 0.3 & 0 \end{bmatrix}$$

is an example of a scattering matrix for a **matched,** three port device.

Lossless

For a lossless device, all of the power that delivered to each device port must eventually find its way **out**!

In other words, power is not **absorbed** by the network—no power to be **converted to heat**!

Recall the **power incident** on some port m is related to the amplitude of the **incident wave** (V_{0m}^{+}) as:

$$P_m^+ = \frac{\left|V_{0m}^+\right|^2}{2Z_0}$$

While power of the **wave exiting** the port is:

$$P_m^- = \frac{\left|V_{0m}^-\right|^2}{2Z_0}$$

Thus, the power **delivered** to (absorbed by) that port is the **difference** of the two:

$$\Delta P_m = P_m^+ - P_m^- = \frac{\left|V_{0m}^+\right|^2}{2Z_0} - \frac{\left|V_{0m}^-\right|^2}{2Z_0}$$

Thus, the **total power incident** on an N-port device is:

$$P^+ = \sum_{m=1}^{N} P_m^+ = \frac{1}{2Z_0}\sum_{m=1}^{N}\left|V_{0m}^+\right|^2$$

Note that:

$$\sum_{m=1}^{N}\left|V_{0m}^+\right|^2 = \left(\mathbf{V}^+\right)^H \mathbf{V}^+$$

where operator H indicates the **conjugate transpose** (i.e., Hermetian transpose) operation, so that $\left(\mathbf{V}^+\right)^H \mathbf{V}^+$ is the **inner product** (i.e., dot product, or scalar product) of complex vector $\mathbf{V}^+$ with itself.

Thus, we can write the **total power incident** on the device as:

$$P^+ = \frac{1}{2Z_0}\sum_{m=1}^{N}\left|V_{0m}^+\right|^2 = \frac{\left(\mathbf{V}^+\right)^H \mathbf{V}^+}{2Z_0}$$

Similarly, we can express the **total power** of the **waves exiting** our M-port network to be:

$$P^- = \frac{1}{2Z_0}\sum_{m=1}^{N}\left|V_{0m}^-\right|^2 = \frac{\left(\mathbf{V}^-\right)^H \mathbf{V}^-}{2Z_0}$$