Filter Realizations Using Lumped Elements

Our **first** filter circuit will be "**realized**" with lumped elements.

Lumped elements—we mean inductors L and capacitors C!

Since each of these elements are (ideally) perfectly **reactive**, the resulting filter will be **lossless** (ideally).

We will first consider two configurations of a **ladder circuit**:

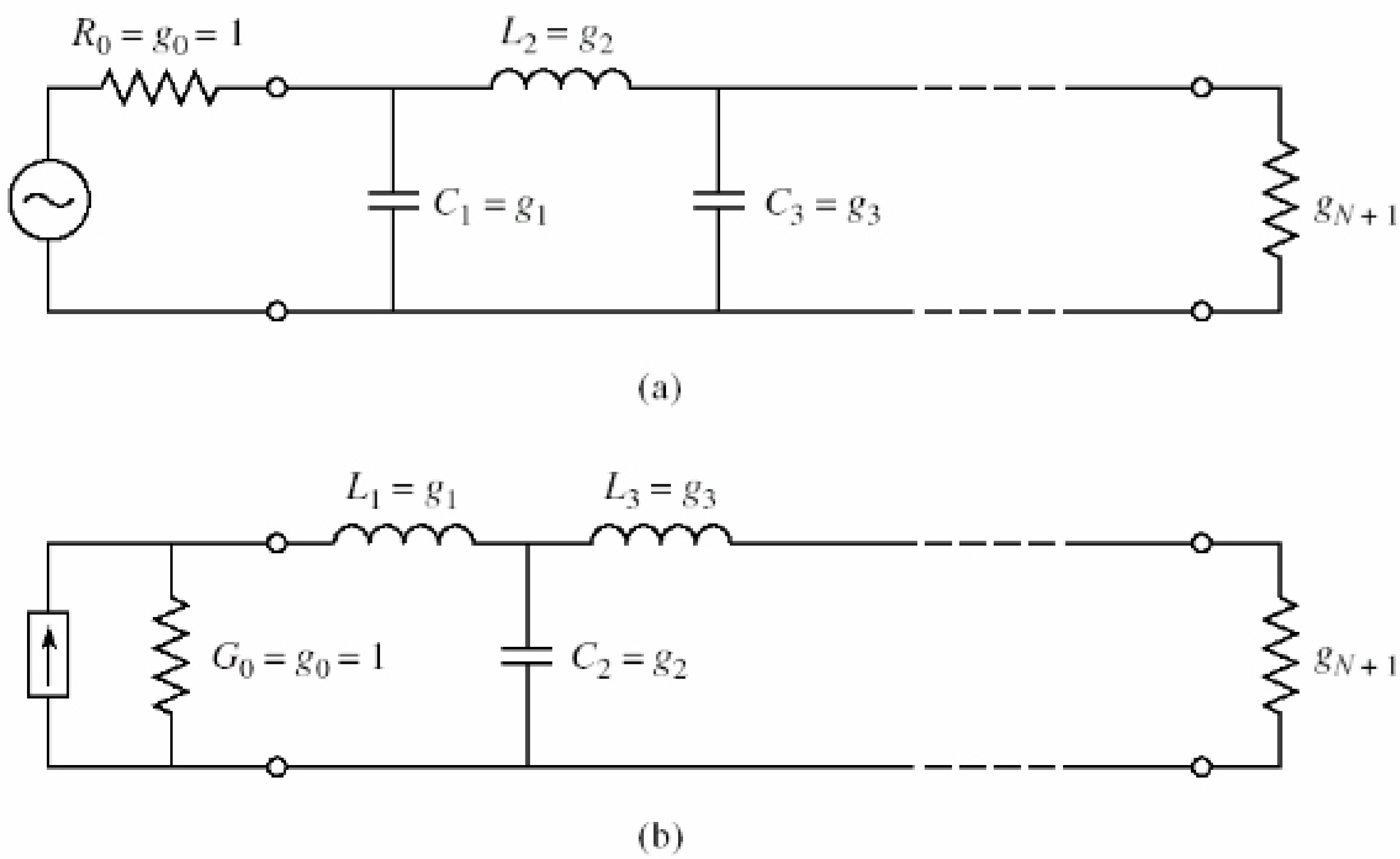

Figure 8.25 (p. 393)

Ladder circuits for low-pass filter prototypes and their element definitions. (*a*) Prototype beginning with a shunt element. (*b*) Prototype beginning with a series element.

Note that these two structures provide a **low-pass** filter response (evaluate the circuits at $\omega = 0$ and $\omega = \infty$!).

Moreover, these structures have N different **reactive elements** (i.e., N degrees of design freedom) and thus can be used to realize an **N-order** power loss ratio.

For example, consider the **Butterworth** power loss ratio function:

$$P_{LR}(\omega) = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N}$$

Recall this is a **low-pass** function, as $P_{LR} = 1$ at $\omega = 0$, and $P_{LR} = \infty$ at $\omega = \infty$. Note also that at $\omega = \omega_c$:

$$P_{LR}(\omega = \omega_c) = 1 + \left(\frac{\omega_c}{\omega_c}\right)^{2N} = 1 + 1^{2N} = 2$$

Meaning that:

$$\Gamma(\omega = \omega_c) = \mathrm{T}(\omega = \omega_c) = \frac{1}{2}$$

In other words, ω_c defines the 3dB bandwidth of the low-pass filter.

Likewise, we find that this Butterworth function is **maximally flat** at $\omega = 0$:

$$P_{LR}(\omega = 0) = 1 + \left(\frac{0}{\omega_c}\right)^{2N} = 1$$

and:

$$\left.\frac{d^n \, P_{LR}(\omega)}{d\omega^n}\right|_{\omega=0} = 0 \qquad \text{for all } n$$

Now, we can determine the function $P_{LR}(\omega)$ for a lumped element ladder circuit of N elements using our knowledge of **complex circuit theory**.

Then, we can **equate** the resulting polynomial to the **maximally flat** function above. In this manner, we can determine the appropriate **values** of all inductors L and capacitors C!

An **example** of this method is given on pages 392 and 393 of your book. In this case, the filter is very **simple**—just **one** inductor and **one** capacitor. However, as the book shows, finding the solution requires quite a bit **complex algebra**!

Fortunately, your book likewise provides **tables** of complete Butterworth and Chebychev Low-Pass **solutions** (up to order 10) for the ladder circuits of figure 8.25—**no** complex algebra required!