Lumped Modeling in Thermal Domain

- ☑ Last lecture
  - Dissipation
  - Thermal dynamics
  - Thermal energy domain
- Today:
  - Lumped modeling
  - Self-heating of a resistor

📖 Reading: Senturia, Chapter 11, p.267-282

Lumped Modeling--Choices

- **First law of thermodynamics for reversible processes:**

$$dU = dQ - dW = TdS - dW$$

- **The work, dW, may also be expressed in terms of the effort and displacement for the coupled energy domains.**

$$dU = TdS - \sum_j e_j dq_j$$

- Temperature and entropy are awkward choices for effort and displacement.
- Use instead temperature difference for across variable and heat current as through variable
  - Note that these are not a conjugate power variable pair.

Thermal Resistance for Heat Conduction

Electrical	Thermal
Voltage V	Temperature T (K)
Charge Q	Heat energy Q (Joules)
Current I	Heat current, $\dot{Q}$ or I_Q (Watts)
Current density J	Heat density, $J_Q = I_Q/A$ (Watts/m^2)
Resistance, R	Thermal resistance, R_T (Kevin/Watts)

From Fourier's Law of heat conduction, $J_Q = -\kappa \nabla T$,

$$\dot{Q} = J_Q A = -\kappa A \nabla T = \kappa A \frac{T_1 - T_2}{L}$$

$$T_1 - T_2 = \left(\frac{L}{\kappa A}\right)\dot{Q} = R_T \dot{Q}$$

where $R_T = \frac{L}{\kappa A}$ [Kelvin/Watts]